Find the location of the indicated absolute extremum for the function.

1) Maximum

A) No maximum  
B) \( x = 0 \)  
C) \( x = 2 \)  
D) \( x = -1 \)

Find the extreme values of the function and where they occur.

2) \( y = \frac{x + 1}{x^2 + 2x + 2} \)

A) The maximum is \( -\frac{1}{2} \) at \( x = 0 \); the minimum is \( \frac{1}{2} \) at \( x = -2 \).

B) The maximum is 2 at \( x = 0 \); the minimum is \( \frac{1}{2} \) at \( x = -2 \).

C) The maximum is \( \frac{1}{2} \) at \( x = 0 \); the minimum is \( -\frac{1}{2} \) at \( x = -2 \).

D) There are none.

Find the derivative at each critical point and determine the local extreme values.

3) \( y = \begin{cases} -x^2 - 4x + 8, & x \leq 1 \\ -x^2 + 12x - 8, & x > 1 \end{cases} \)

<table>
<thead>
<tr>
<th>Critical Pt</th>
<th>Derivative</th>
<th>Extremum</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = -2 )</td>
<td>0</td>
<td>local max</td>
<td>12</td>
</tr>
<tr>
<td>( x = 1 )</td>
<td>undefined</td>
<td>local min</td>
<td>5</td>
</tr>
<tr>
<td>( x = -6 )</td>
<td>0</td>
<td>local max</td>
<td>28</td>
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B) 

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<td>0</td>
<td>local max</td>
<td>28</td>
</tr>
</tbody>
</table>
C) Critical Pt. | Derivative | Extremum | Value
--- | --- | --- | ---
-2 | 0 | local min | 12
1 | undefined | local max | 3
6 | 0 | local min | 28

D) Critical Pt. | Derivative | Extremum | Value
--- | --- | --- | ---
2 | 0 | local max | 12
1 | undefined | local min | 5
6 | undefined | local max | 28

Determine whether the function satisfies the hypotheses of the Mean Value Theorem on the given interval.

4) \( f(x) = \begin{cases} \cos 8x, & -\pi \leq x < 0 \\ x, & 0 \leq x \leq \pi \end{cases} \) on \([-\pi, 0]\)

A) No
B) Yes

Give an appropriate answer.

5) Find the value or values of \( c \) that satisfy \( \frac{f(b) - f(a)}{b - a} = f'(c) \) for the function \( f(x) = x + \frac{27}{x} \) on the interval \([3, 9]\).

A) \( 3\sqrt{3} \)
B) 3, 9
C) 0, \( 3\sqrt{3} \)
D) \(-3\sqrt{3}, 3\sqrt{3}\)

Answer the question.

6) A marathoner ran the 26.2 mile New York City Marathon in 2.3 hrs. Did the runner ever exceed a speed of 9 miles per hour?

A marathoner ran the 26.2 mile New York City Marathon in 2.3 hrs. Did the runner ever exceed a speed of 9 miles per hour?

Use analytic methods to find the local extrema.

7) \( f(x) = (x - 6)^3; (-\infty, \infty) \)

A) Local minimum: 0
B) Local minimum: 0; local maximum: 6
C) Local minimum: 6
D) No local extrema

Use analytic methods to find those values of \( x \) for which the given function is increasing and those values of \( x \) for which it is decreasing.

8) \( f(x) = x^4 - 2 \)

A) Increasing on \((-\infty, -1)\) and \((1, \infty)\), decreasing on \((-1, 1)\)
B) Increasing on \((-1, 1)\), decreasing on \((-\infty, -1)\) and \((1, \infty)\)
C) Increasing on \((-\infty, -1)\) and \((0, 1)\), decreasing on \((-1, 0)\) and \((1, \infty)\)
D) Increasing on \((-1, 0)\) and \((1, \infty)\), decreasing on \((-\infty, -1)\) and \((0, 1)\)
Find all possible functions with the given derivative.
9) \( f'(x) = 4 \cos 4x \)
   A) \( \sin 4x + C \) \hspace{1cm} B) \( \cos x + C \) \hspace{1cm} C) \( \cos 4x + C \) \hspace{1cm} D) \( \sin x + C \)

Find the function with the given derivative whose graph passes through the point \( P \).
10) \( f'(x) = x^2 + 9 \), \( P(3, 55) \)
    A) \( f(x) = \frac{x^3}{3} + 9x \) \hspace{1cm} B) \( f(x) = x^3 + 9x + 1 \)
    C) \( f(x) = \frac{x^3}{3} + 9x + 19 \) \hspace{1cm} D) \( f(x) = x^3 + 9x^2 + 19 \)

Sketch a graph of a function \( y = f(x) \) that has the given properties.
11) a) Differentiable everywhere except \( x = 0 \)
    b) Continuous for all real numbers
    c) \( f(x) < 0 \) on \( (-\infty, 0) \)
    d) \( f(x) > 0 \) on \( (0, \infty) \)
    e) \( f(-2) = f(2) = 5 \)
    f) \( y \)-intercept and \( x \)-intercept at \( (0, 0) \)

Use the First Derivative Test to determine the local extrema of the function, and identify any absolute extrema.
12) \( f(x) = -x \sqrt[9]{9 - x^2} \)
    A) Local maximum \( \left( \frac{9}{2}, \frac{9}{2} \right) \)
    B) Local minimum \( (0, 0) \)
    C) Absolute minimum \( \left( \frac{9}{2}, -\frac{9}{2} \right) \), absolute maximum \( \left( -\frac{9}{2}, \frac{9}{2} \right) \)
    D) Absolute maximum \( \left( \frac{9}{2}, \frac{9}{2} \right) \), absolute minimum \( \left( -\frac{9}{2}, -\frac{9}{2} \right) \)

Use the Concavity Test to find the intervals where the graph of the function is concave up.
13) \( y = 6x - 6e^{-x} \)
    A) \( (-\infty, \infty) \) \hspace{1cm} B) \( (-\infty, 0) \) \hspace{1cm} C) \( (0, \infty) \) \hspace{1cm} D) None

Find the points of inflection.
14) \( y = x \sqrt[11]{11 - x^2} \)
    A) \( (0, 11) \) \hspace{1cm} B) \( (0, 0) \)
    C) No inflection points. \hspace{1cm} D) \( (11, 0) \)
Use the graph of \( f \) to estimate where \( f' \) is 0, positive, and negative.

A) Zero: \( x = \pm 1 \); positive: \( x = (1, \infty) \); negative: \( x = (-1, 1) \)
B) Zero: \( x = \pm 1 \); positive: \( x = (-\infty, -1) \); negative: \( x = (-1, 1) \)
C) Zero: \( x = \pm 1 \); positive: \( x = (-\infty, -1) \) and \( (1, \infty) \); negative: \( x = (-1, 1) \)
D) Zero: \( x = \pm 1 \); positive: \( x = (-\infty, -1) \) and \( (1, \infty) \); negative: \( x = (0, 1) \)

Use the Second Derivative Test to find the local extrema for the function.

16) \( f(x) = x^3 + 12x^2 + 48x - 4 \)
   A) No local extrema
   B) Local minimum at \( x = -4 \)
   C) Local maximum at \( x = -4 \); local minimum at \( x = 4 \)
   D) Local maximum at \( x = -4 \)

Use the given derivative of the function to find the local extrema of the function.

17) \( y' = (x - 9)^2(x + 8) \)
   A) Local maximum at \( x = -8 \)
   B) Local minimum at \( x = -8 \)
   C) Local minimum at \( x = 9 \)
   D) None

Solve the problem.

18) The graphs below show the first and second derivatives of a function \( y = f(x) \). Select a possible graph \( f \) that passes through the point \( P \).
19) Select an appropriate graph of a twice-differentiable function $y = f(x)$ that passes through the points $(-\sqrt{2}, 1), \left(-\frac{\sqrt{6}}{3}, \frac{5}{9}\right), (0,0), \left(\frac{\sqrt{6}}{3}, \frac{5}{9}\right)$, and $(\sqrt{2}, 1)$, and whose first two derivatives have the following sign patterns.

\[
y' : \begin{array}{cccc}
+ & - & + & - \\
-\sqrt{2} & 0 & \sqrt{2} & \\
\end{array}
\]

\[
y'' : \begin{array}{cccc}
- & + & - & \\
\frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{3} & \\
\end{array}
\]

Solve the problem analytically.

20) Of all numbers whose difference is 12, find the two that have the minimum product.

A) 1 and 13  
B) 0 and 12  
C) 24 and 12  
D) 6 and 6

Solve the problem.

21) A company is constructing an open-top, square-based, rectangular metal tank that will have a volume of 27 ft$^3$. What dimensions yield the minimum surface area? Round to the nearest tenth, if necessary.

A) 7.3 ft $\times$ 7.3 ft $\times$ 0.5 ft  
B) 3.8 ft $\times$ 3.8 ft $\times$ 1.9 ft  
C) 4.3 ft $\times$ 4.3 ft $\times$ 1.4 ft  
D) 3 ft $\times$ 3 ft $\times$ 3 ft

22) Suppose $c(x) = x^3 - 18x^2 + 10,000x$ is the cost of manufacturing $x$ items. Find a production level that will minimize the average cost of making $x$ items.

A) 10 items  
B) 9 items  
C) 8 items  
D) 11 items
Find the linearization $L(x)$ of $f(x)$ at $x = a$.

23) $f(x) = \frac{3}{4}x + \frac{2}{3}$, $a = 2$

A) $L(x) = \frac{3}{4}x + \frac{2}{3}$
B) $L(x) = \frac{5}{4}x + 1$
C) $L(x) = \frac{5}{4}x + \frac{2}{3}$
D) $L(x) = \frac{3}{4}x + 1$

Use the linearization $(1 + x)^k \approx 1 + kx$ to approximate the value. Give your answer in the form indicated.

24) $\sqrt[3]{1.012}$

Give your answer as a decimal.
A) 1.012
B) 2.004
C) 1.004
D) 1.04

Use the linear approximation $(1 + x)^k \approx 1 + kx$, as specified.

25) Find an approximation for the function $f(x) = \frac{1}{\sqrt{5} + x}$ for values of $x$ near zero.

A) $f(x) \approx -2 + \frac{1}{2}x$
B) $f(x) \approx \frac{1}{\sqrt{5}}[1 - \frac{x}{10}]$
C) $f(x) \approx -2 + \frac{1}{2}x$
D) $f(x) \approx 1 - \frac{1}{2}x$

Approximate the root by using a linearization centered at an appropriate nearby number.

26) $\sqrt[3]{33}$

A) 3.3222
B) 3.2222
C) 3.4222
D) 3.1222

Find the differential.

27) $y = \sqrt{9x - 4}$

A) $\frac{27x + 8}{\sqrt{9x - 4}}$ dx
B) $\frac{27x - 8}{\sqrt{9x - 4}}$ dx
C) $\frac{27x + 8}{2\sqrt{9x - 4}}$ dx
D) $\frac{27x - 8}{2\sqrt{9x - 4}}$ dx

Find the differential.

28) $d(\csc(5x^2 - 1))$

A) $10x \csc(5x^2 - 1) \cot(5x^2 - 1) dx$
B) $-10x \csc(10x) \cot(10x) dx$
C) $-5x^2 \csc(5x^2 - 1) \cot(5x^2 - 1) dx$
D) $-10x \csc(5x^2 - 1) \cot(5x^2 - 1) dx$

Solve the problem.

29) A rectangular swimming pool 16 m by 12 m is being filled at the rate of 0.9 m$^3$/min. How fast is the height $h$ of the water rising?

A) 0.0047 m/min
B) 173 m/min
C) 0.97 m/min
D) 0.30 m/min

30) One airplane is approaching an airport from the north at 124 km/hr. A second airplane approaches from the east at 223 km/hr. Find the rate at which the distance between the planes changes when the southbound plane is 34 km away from the airport and the westbound plane is 15 km from the airport.

A) 335 km/hr
B) 1247 km/hr
C) 98 km/hr
D) 1411 km/hr