11.2 Simplifying Radical Expressions

Essential Question: How can you simplify expressions containing rational exponents or radicals involving *n*th roots?

Explore Establishing the Properties of Rational Exponents

In previous courses, you have used properties of integer exponents to simplify and evaluate expressions, as shown here for a few simple examples:

$4^2 \cdot 4^3 = 4^{2+3} = 4^5 = 1024$

$(4 \cdot x)^2 = 4^2 \cdot x^2 = 16x^2$

$(4^2)^3 = 4^{2 \cdot 3} = 4^6 = 4096$

$\frac{4^2}{4^3} = 4^{2-3} = 4^{-1} = \frac{1}{4}$

$\left(\frac{4}{x}\right)^3 = \frac{4^3}{x^3} = \frac{64}{x^3}$

Now that you have been introduced to expressions involving rational exponents, you can explore the properties that apply to simplifying them.

(A) Let $a = 64$, $b = 4$, $m = \frac{1}{3}$, and $n = \frac{3}{2}$. Evaluate each expression by substituting and applying exponents individually, as shown.

Expression	Substitute	Simplify	Result
$a^m \cdot a^n$	$64^{\frac{1}{3}} \cdot 64^{\frac{3}{2}}$	$4 \cdot 512$	2048
$(a \cdot b)^n$	$(64 \cdot 4)^{\frac{3}{2}}$	$256^{\frac{3}{2}}$	
$(a^m)^n$			
$\frac{a^n}{a^m}$			
$\left(\frac{a}{b}\right)^n$			

Expression	Substitute	Simplify	Result
$a^m \cdot a^n$	$64^{\frac{1}{3}} \cdot 64^{\frac{3}{2}}$	$4 \cdot 512$	2048
$(a \cdot b)^n$	$(64 \cdot 4)^{\frac{3}{2}}$	$256^{\frac{3}{2}}$	**4096**
$(a^m)^n$	$\left(64^{\frac{1}{3}}\right)^{\frac{3}{2}}$	$4^{\frac{3}{2}}$	**8**
$\frac{a^n}{a^m}$	$\frac{64^{\frac{3}{2}}}{64^{\frac{1}{3}}}$	$\frac{512}{4}$	**128**
$\left(\frac{a}{b}\right)^n$	$\left(\frac{64}{4}\right)^{\frac{3}{2}}$	$16^{\frac{3}{2}}$	**64**

Essential Question: How can you simplify expressions containing rational exponents or radicals involving nth roots?

Possible answer: You can use the same properties of exponents for rational exponents as for integer exponents, apply the properties of square roots to radicals involving *n*th roots, and translate between radical form and rational exponent form whenever it is helpful.

Ⓑ Complete the table again. This time, however, apply the rule of exponents that you would use for integer exponents.

Expression	Apply Rule and Substitute	Simplify	Result
$a^m \cdot a^n$	$64^{\frac{1}{3}+\frac{3}{2}}$	$64^{\frac{11}{6}}$	
$(a \cdot b)^n$			
$(a^m)^n$			
$\frac{a^n}{a^m}$			
$\left(\frac{a}{b}\right)^n$			

Expression	Apply Rule and Substitute	Simplify	Result
$a^m \cdot a^n$	$64^{\frac{1}{3}+\frac{3}{2}}$	$64^{\frac{11}{6}}$	**2048**
$(a \cdot b)^n$	$64^{\frac{3}{2}} \cdot 4^{\frac{3}{2}}$	$512 \cdot 8$	**4096**
$(a^m)^n$	$64^{\frac{1}{3} \cdot \frac{3}{2}}$	$64^{\frac{1}{2}}$	**8**
$\frac{a^n}{a^m}$	$64^{\frac{3}{2}-\frac{1}{3}}$	$64^{\frac{7}{6}}$	**128**
$\left(\frac{a}{b}\right)^n$	$\frac{64^{\frac{3}{2}}}{4^{\frac{3}{2}}}$	$\frac{512}{8}$	**64**

Reflect

1. Compare your results in Steps A and B. What can you conclude?

2. In Steps A and B, you evaluated $\frac{a^n}{a^m}$ two ways. Now evaluate $\frac{a^m}{a^n}$ two ways, using the definition of negative exponents. Are your results consistent with your previous conclusions about integer and rational exponents?

Reflect

1. Compare your results in Steps A and B. What can you conclude?

Applying the same rules as for integer exponents gives the same results as applying the exponents individually. The properties of rational exponents are the same as the corresponding properties of integer exponents.

2. In Steps A and B, you evaluated $\frac{a^n}{a^m}$ two ways. Now evaluate $\frac{a^m}{a^n}$ two ways, using the definition of negative exponents. Are your results consistent with your previous conclusions about integer and rational exponents?

$\frac{a^m}{a^n} = \frac{64^{\frac{1}{3}}}{64^{\frac{3}{2}}} = \frac{4}{512} = \frac{1}{128}$;

$\frac{a^m}{a^n} = 64^{\frac{1}{3} - \frac{3}{2}} = 64^{-\frac{7}{6}} = \frac{1}{64^{\frac{7}{6}}} = \frac{1}{128}$;

Yes, working with negative rational exponents is consistent with working with negative integer exponents.

QUESTIONING STRATEGIES

? If an expression consists of a variable raised to a negative exponent, how can you rewrite the expression with a positive exponent? **Rewrite the expression as the reciprocal of the variable raised to the opposite of the exponent.**

? How does that help you write the simplified form of $\frac{x^{\frac{1}{4}}}{x^{\frac{3}{4}}}$ with a positive exponent? **You can subtract the exponents, and then apply the rule to the answer.** $\frac{x^{\frac{1}{4}}}{x^{\frac{3}{4}}} = x^{\frac{1}{4} - \frac{3}{4}} = x^{-\frac{1}{2}} = \left(\frac{1}{x}\right)^{\frac{1}{2}}$

Explain 1 Simplifying Rational-Exponent Expressions

Rational exponents have the same properties as integer exponents.

Properties of Rational Exponents

For all nonzero real numbers *a* and *b* and rational numbers *m* and *n*

Words	Numbers	Algebra
Product of Powers Property To multiply powers with the same base, add the exponents.	$12^{\frac{1}{2}} \cdot 12^{\frac{3}{2}} = 12^{\frac{1}{2}+\frac{3}{2}} = 12^2 = 144$	$a^m \cdot a^n = a^{m+n}$
Quotient of Powers Property To divide powers with the same base, subtract the exponents.	$\frac{125^{\frac{2}{3}}}{125^{\frac{1}{3}}} = 125^{\frac{2}{3}-\frac{1}{3}} = 125^{\frac{1}{3}} = 5$	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power Property To raise one power to another, multiply the exponents.	$\left(8^{\frac{2}{3}}\right)^3 = 8^{\frac{2}{3}\cdot 3} = 8^2 = 64$	$(a^m)^n = a^{m \cdot n}$
Power of a Product Property To find a power of a product, distribute the exponent.	$(16 \cdot 25)^{\frac{1}{2}} = 16^{\frac{1}{2}} \cdot 25^{\frac{1}{2}} = 4 \cdot 5 = 20$	$(ab)^m = a^m b^m$
Power of a Quotient Property To find the power of a qoutient, distribute the exponent.	$\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{16^{\frac{1}{4}}}{81^{\frac{1}{4}}} = \frac{2}{3}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Example 1 **Simplify the expression. Assume that all variables are positive. Exponents in simplified form should all be positive.**

(A) **a.** $25^{\frac{3}{5}} \cdot 25^{\frac{7}{5}}$

$= 25^{\frac{3}{5}+\frac{7}{5}}$ Product of Powers Prop.

$= 25^2$ Simplify.

$= 625$

b. $\frac{8^{\frac{1}{3}}}{8^{\frac{2}{3}}}$

$= 8^{\frac{1}{3}-\frac{2}{3}}$ Quotient of Powes Prop.

$= 8^{-\frac{1}{3}}$ Simplify.

$= \frac{1}{8^{\frac{1}{3}}}$ Definition of neg. power

$= \frac{1}{2}$ Simplify.

How do you multiply powers with the same base when the exponents are rational? **Add the exponents, and write the result as a power of the common base.**

How do you divide powers with the same base when the exponents are rational? **Subtract the exponents, and write the result as a power of the common base.**

(B) **a.** $\left(\frac{y^{\frac{4}{3}}}{16y^{\frac{2}{3}}}\right)^{\frac{3}{2}}$

$= \left(\frac{y^{\frac{4}{3}-\frac{2}{3}}}{16}\right)^{\frac{3}{2}}$ ______ Prop.

$= \left(\frac{\square}{16}\right)^{\frac{3}{2}}$ Simplify.

$= \frac{\left(y^{\frac{2}{3}}\right)^{\frac{3}{2}}}{16^{\frac{3}{2}}}$ ______ Prop.

$= \frac{y^{\frac{2}{3}\cdot\frac{3}{2}}}{16^{\frac{3}{2}}}$ ______ Prop.

$= \square$ Simplify.

b. $\left(27x^{\frac{3}{4}}\right)^{\frac{2}{3}}$

$= \square^{\frac{2}{3}}\left(\square\right)^{\frac{2}{3}}$ Power of a Product Prop.

$= 27^{\frac{2}{3}}\left(x^{\square}\right)$ Power of a Power Prop.

$= \square$ Simplify.

(B) **a.** $\left(\dfrac{y^{\frac{4}{3}}}{16y^{\frac{2}{3}}}\right)^{\frac{3}{2}}$

$= \left(\dfrac{y^{\frac{4}{3}-\frac{2}{3}}}{16}\right)^{\frac{3}{2}}$ **Quotient of Powers** Prop.

$= \left(\dfrac{y^{\frac{2}{3}}}{16}\right)^{\frac{3}{2}}$ Simplify.

$= \dfrac{\left(y^{\frac{2}{3}}\right)^{\frac{3}{2}}}{16^{\frac{3}{2}}}$ **Power of a Quotient** Prop.

$= \dfrac{y^{\frac{2}{3}\cdot\frac{3}{2}}}{16^{\frac{3}{2}}}$ **Power of a power** Prop.

$= \dfrac{y}{64}$ Simplify.

b. $\left(27x^{\frac{3}{4}}\right)^{\frac{2}{3}}$

$= 27^{\frac{2}{3}}\left(x^{\frac{3}{4}}\right)^{\frac{2}{3}}$ Power of a Product Prop.

$= 27^{\frac{2}{3}}\left(x^{\frac{3}{4}\cdot\frac{2}{3}}\right)$ Power of a Power Prop.

$= 9x^{\frac{1}{2}}$ Simplify.

Your Turn

Simplify the expression. Assume that all variables are positive. Exponents in simplified form should all be positive.

3. $\left(12^{\frac{2}{3}} \cdot 12^{\frac{4}{3}}\right)^{\frac{3}{2}}$

4. $\dfrac{\left(6x^{\frac{1}{3}}\right)^2}{x^{\frac{5}{3}}y}$

Your Turn

Simplify the expression. Assume that all variables are positive. Exponents in simplified form should all be positive.

3. $\left(12^{\frac{2}{3}} \cdot 12^{\frac{4}{3}}\right)^{\frac{3}{2}}$

$\left(12^{\frac{2}{3}} \cdot 12^{\frac{4}{3}}\right)^{\frac{3}{2}} = \left(12^{\frac{2}{3}+\frac{4}{3}}\right)^{\frac{3}{2}} = \left(12^2\right)^{\frac{3}{2}}$

$= 12^{2\cdot\frac{3}{2}} = 12^3 = 1728$

4. $\dfrac{\left(6x^{\frac{1}{3}}\right)^2}{x^{\frac{5}{3}}y}$

$\dfrac{\left(6x^{\frac{1}{3}}\right)^2}{x^{\frac{5}{3}}y} = \dfrac{6^2x^{\frac{1}{3}\cdot 2}}{x^{\frac{5}{3}}y} = \dfrac{36x^{\frac{2}{3}}}{x^{\frac{5}{3}}y} = \dfrac{36x^{\frac{2}{3}-\frac{5}{3}}}{y}$

$= \dfrac{36x^{-1}}{y} = \dfrac{36}{xy}$

Explain 2 Simplifying Radical Expressions Using the Properties of Exponents

When you are working with radical expressions involving nth roots, you can rewrite the expressions using rational exponents and then simplify them using the properties of exponents.

Example 2 **Simplify the expression by writing it using rational exponents and then using the properties of rational exponents. Assume that all variables are positive. Exponents in simplified form should all be positive.**

(A) $x\left(\sqrt[3]{2y}\right)\left(\sqrt[3]{4x^2y^2}\right)$

$= x(2y)^{\frac{1}{3}}\left(4x^2y^2\right)^{\frac{1}{3}}$ Write using rational exponents.

$= x\left(2y \cdot 4x^2y^2\right)^{\frac{1}{3}}$ Power of a Product Property

$= x\left(8x^2y^3\right)^{\frac{1}{3}}$ Product of Powers Property

$= x\left(2x^{\frac{2}{3}}y\right)$ Power of a Product Property

$= 2x^{\frac{5}{3}}y$ Product of Powers Property

(B) $\dfrac{\sqrt{64y}}{\sqrt[3]{64y}}$

$= \dfrac{(64y)^{\frac{1}{2}}}{(64y)^{\frac{1}{3}}}$ Write using rational exponents.

$= (64y)^{\square}$ Quotient of Powers Property

$= (64y)^{\square}$ Simplify.

$= \square$ Power of a Product Property

$= \square$ Simplify.

(B) $\dfrac{\sqrt{64y}}{\sqrt[3]{64y}}$

$= \dfrac{(64y)^{\frac{1}{2}}}{(64y)^{\frac{1}{3}}}$ Write using rational exponents.

$= (64y)^{\frac{1}{2}-\frac{1}{3}}$ Quotient of Powers Property

$= (64y)^{\frac{1}{6}}$ Simplify.

$= 64^{\frac{1}{6}}y^{\frac{1}{6}}$ Power of a Product Property

$= 2y^{\frac{1}{6}}$ Simplify.

AVOID COMMON ERRORS

After learning the product and quotient properties for nth roots, students may assume there are similar properties for sums and differences. Show students, by numerical example, that $\sqrt[n]{a+b} \neq \sqrt[n]{a} + \sqrt[n]{b}$ and $\sqrt[n]{a-b} \neq \sqrt[n]{a} - \sqrt[n]{b}$ for $a, b > 0$.

Explain 3 Simplifying Radical Expressions Using the Properties of n^{th} Roots

From working with square roots, you know, for example, that $\sqrt{8} \cdot \sqrt{2} = \sqrt{8 \cdot 2} = \sqrt{16} = 4$ and $\frac{\sqrt{8}}{\sqrt{2}} \cdot = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$. The corresponding properties also apply to nth roots.

Properties of nth Roots

For a > 0 and b > 0

Words	Numbers	Algebra
Product Property of Roots The nth root of a product is equal to the product of the nth roots.	$\sqrt[3]{16} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
Quotient Property of Roots The nth root of a Quotient is equal to the Quotient of the nth roots.	$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Example 3 **Simplify the expression using the properties of nth roots. Assume that all variables are positive. Rationalize any irrational denominators.**

(A) $\sqrt[3]{256x^3y^7}$

$\sqrt[3]{256x^3y^7}$

$= \sqrt[3]{2^8 \cdot x^3y^7}$ Write 256 as a power.

$= \sqrt[3]{2^6 \cdot x^3y^6} \cdot \sqrt[3]{2^2 \cdot y}$ Product Property of Roots

$= \sqrt[3]{2^6} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{y^6} \cdot \sqrt[3]{4y}$ Factor out perfect cubes.

$= 4xy^2\sqrt[3]{4y}$ Simplify.

Ⓑ $\sqrt[4]{\frac{81}{x}}$

$\sqrt[4]{\frac{81}{x}}$

$= \frac{\sqrt[4]{81}}{\sqrt[4]{x}}$ ☐

$= \frac{☐}{\sqrt[4]{x}}$ Simplify.

$= \frac{3}{\sqrt[4]{x}} \cdot \frac{☐}{\sqrt[4]{x^3}}$ Rationalize the denominator.

$= \frac{3\sqrt[4]{x^3}}{\sqrt[4]{x^4}}$ ☐

$=$ ☐ Simplify.

Ⓑ $\sqrt[4]{\frac{81}{x}}$

$\sqrt[4]{\frac{81}{x}}$

$= \frac{\sqrt[4]{81}}{\sqrt[4]{x}}$ **Quotient Property of Roots**

$= \frac{\mathbf{3}}{\sqrt[4]{x}}$ Simplify.

$= \frac{3}{\sqrt[4]{x}} \cdot \frac{\sqrt[4]{x^3}}{\sqrt[4]{x^3}}$ Rationalize the denominator.

$= \frac{3\sqrt[4]{x^3}}{\sqrt[4]{x^4}}$ **Product Property of Roots**

$= \frac{3\sqrt[4]{x^3}}{x}$ Simplify.

Reflect

7. In Part B, why was $\sqrt[4]{x^3}$ used when rationalizing the denominator? What factor would you use to rationalize a denominator of $\sqrt[5]{4y^3}$?

Reflect

7. In Part B, why was $\sqrt[4]{x^3}$ used when rationalizing the denominator? What factor would you use to rationalize a denominator of $\sqrt[5]{4y^3}$?

It was chosen to make the product of the radicands be a perfect fourth power so that the fourth root could be taken; $\sqrt[5]{8y^2}$.

Your Turn

Simplify the expression using the properties of *n*th roots. Assume that all variables are positive.

8. $\sqrt[3]{216x^{12}y^{15}}$

9. $\sqrt[4]{\frac{16}{x^{14}}}$

Your Turn

Simplify the expression using the properties of *n*th roots. Assume that all variables are positive.

8. $\sqrt[3]{216x^{12}y^{15}}$

$\sqrt[3]{216x^{12}y^{15}} = \sqrt[3]{6^3} \cdot \sqrt[3]{x^{12}} \cdot \sqrt[3]{y^{15}} = 6x^4y^5$

9. $\sqrt[4]{\frac{16}{x^{14}}}$

$= \frac{\sqrt[4]{16}}{\sqrt[4]{x^{14}}}$

$= \frac{2}{\sqrt[4]{x^{12}} \cdot \sqrt[4]{x^2}}$

$= \frac{2}{\sqrt[4]{x^{12}}} \cdot \frac{1}{\sqrt[4]{x^2}}$

$= \frac{2}{x^3} \cdot \frac{1}{\sqrt[4]{x^2}} \cdot \frac{\sqrt[4]{x^2}}{\sqrt[4]{x^2}}$

$= \frac{2\sqrt[4]{x^2}}{x^4}$

Explain 4 Rewriting a Radical-Function Model

When you find or apply a function model involving rational powers or radicals, you can use the properties in this lesson to help you find a simpler expression for the model.

(A) **Manufacturing** A can that is twice as tall as its radius has the minimum surface area for the volume it contains. The formula $S = 6\pi\left(\frac{V}{2\pi}\right)^{\frac{2}{3}}$ expresses the surface area of a can with this shape in terms of its volume.

a. Use the properties of rational exponents to simplify the expression for the surface area. Then write the approximate model with the coefficient rounded to the nearest hundredth.

If you have a graphing calculator

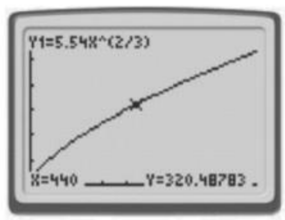
b. Graph the model using a graphing calculator. What is the surface area in square centimeters for a can with a volume of 440 cm^3?

a.

	$S = 6\pi\left(\frac{V}{2\pi}\right)^{\frac{2}{3}}$
Power of a Quotient Property	$= 6\pi \cdot \frac{V^{\frac{2}{3}}}{(2\pi)^{\frac{2}{3}}}$
Group Powers of 2π.	$= \frac{3(2\pi)}{(2\pi)^{\frac{2}{3}}} \cdot V^{\frac{2}{3}}$
Quotient of Powers Property	$= 3(2\pi)^{1-\frac{2}{3}} \cdot V^{\frac{2}{3}}$
Simplify.	$= 3(2\pi)^{\frac{1}{3}} \cdot V^{\frac{2}{3}}$
Use a calculator.	$\approx 5.54V^{\frac{2}{3}}$

A simplified model is $S = 3(2\pi)^{\frac{1}{3}} \cdot V^{\frac{2}{3}}$, which gives $S \approx 5.54V^{\frac{2}{3}}$.

b.



The surface area is about 320 cm^2.

Ⓑ **Commercial fishing** The buoyancy of a fishing float in water depends on the volume of air it contains. The radius of a spherical float as a function of its volume is given by $r = \sqrt[3]{\frac{3V}{4\pi}}$.

a. Use the properties of roots to rewrite the expression for the radius as the product of a coefficient term and a variable term. Then write the approximate formula with the coefficient rounded to the nearest hundredth.

b. What should the radius be for a float that needs to contain 4.4 ft^3 of air to have the proper buoyancy?

a.

	$r = \sqrt[3]{\frac{3V}{4\pi}}$
Rewrite radicand.	$= \sqrt[3]{\frac{3}{4\pi} \cdot \square}$
Product Property of Roots	$= \sqrt[3]{\frac{3}{4\pi}} \cdot \square$
Use a calculator	$\approx \square$

The rewritten formula is $r = \square$, which gives $r \approx \square$.

b. Substitute 4.4 for V.

$r = 0.62\sqrt[3]{4.4} \approx \square$

The radius is about ____ feet.

a. $r = \sqrt[3]{\frac{3V}{4\pi}}$

Rewrite radicand. $= \sqrt[3]{\frac{3}{4\pi} \cdot \boxed{V}}$

Product Property of Roots $= \sqrt[3]{\frac{3}{4\pi}} \cdot \boxed{\sqrt[3]{V}}$

Use a calculator $\approx \boxed{0.62\sqrt[3]{V}}$

The rewritten formula is $r = \boxed{\sqrt[3]{\frac{3}{4\pi}} \cdot \sqrt[3]{V}}$, which gives $r \approx \boxed{0.62\sqrt[3]{V}}$.

b. Substitute 4.4 for V.

$r = 0.62\sqrt[3]{4.4} \approx \boxed{1.02}$

The radius is about 1.0 feet.

Reflect

10. Discussion What are some reasons you might want to rewrite an expression involving radicals into an expression involving rational exponents?

Reflect

10. Discussion What are some reasons you might want to rewrite an expression involving radicals into an expression involving rational exponents?

By rewriting radical expressions, especially complicated ones and those involving *n*th roots and powers in the radicand, rewriting using rational exponents lets you use the properties of rational exponents to make simplification easier. Also, rational exponents make it easier to enter an expression into a calculator for evaluation or graphing.

Your Turn

11. The surface area as a function of volume for a box with a square base and a height that is twice the side length of the base is $S = 10\left(\frac{V}{2}\right)^{\frac{2}{3}}$. Use the properties of rational exponents to simplify the expression for the surface area so that no fractions are involved. Then write the approximate model with the coefficient rounded to the nearest hundredth.

Your Turn

11. The surface area as a function of volume for a box with a square base and a height that is twice the side length of the base is $S = 10\left(\frac{V}{2}\right)^{\frac{2}{3}}$. Use the properties of rational exponents to simplify the expression for the surface area so that no fractions are involved. Then write the approximate model with the coefficient rounded to the nearest hundredth.

$$S = 10\left(\frac{V}{2}\right)^{\frac{2}{3}} = 10 \cdot \frac{V^{\frac{2}{3}}}{2^{\frac{2}{3}}} = \frac{5 \cdot 2}{2^{\frac{2}{3}}} \cdot V^{\frac{2}{3}} = 5\left(2^{1-\frac{2}{3}}\right)V^{\frac{2}{3}} = 5 \cdot 2^{\frac{1}{3}}V^{\frac{2}{3}} \approx 6.30V^{\frac{2}{3}}$$

The expression is $S = 5 \cdot 2^{\frac{1}{3}}V^{\frac{2}{3}}$, which gives $S = 6.30V^{\frac{2}{3}}$.

Elaborate

12. In problems with a radical in the denominator, you rationalized the denominator to remove the radical. What can you do to remove a rational exponent from the denominator? Explain by giving an example.

12. In problems with a radical in the denominator, you rationalized the denominator to remove the radical. What can you do to remove a rational exponent from the denominator? Explain by giving an example.

You can multiply the expression by a form of 1 so that the denominator of the resulting expression has an exponent that is an integer. For example, for $\frac{6}{x^{\frac{2}{5}}}$, multiply the numerator and denominator by $x^{\frac{3}{5}}$: $\frac{6}{x^{\frac{2}{5}}} \cdot \frac{x^{\frac{3}{5}}}{x^{\frac{3}{5}}} = \frac{6x^{\frac{3}{5}}}{x^{\frac{2}{5}+\frac{3}{5}}} = \frac{6x^{\frac{3}{5}}}{x}$.

13. Show why $\sqrt[n]{a^n}$ is equal to a for all natural numbers a and n using the definition of nth roots and using rational exponents.

14. Show that the Product Property of Roots is true using rational exponents.

15. **Essential Question Check-In** Describe the difference between applying the Power of a Power Property and applying the Power of a Product Property for rational exponents using an example that involves both properties.

13. Show why $\sqrt[n]{a^n}$ is equal to a for all natural numbers a and n using the definition of nth roots and using rational exponents.

By definition, the nth root of a number b is the number whose nth power is b. So the nth root of a^n is the number whose nth power is a^n, or a. Using rational exponents, the nth root is indicated by the exponent $\frac{1}{n}$, so $\sqrt[n]{a^n} = (a^n)^{\frac{1}{n}} = a^{n \cdot \frac{1}{n}} = a^{\frac{n}{n}} = a^1 = a$.

14. Show that the Product Property of Roots is true using rational exponents.

The nth root is indicated by the exponent $\frac{1}{n}$, so $\sqrt[n]{ab} = (ab)^{\frac{1}{n}} = a^{\frac{1}{n}} \cdot b^{\frac{1}{n}} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.

15. **Essential Question Check-In** Describe the difference between applying the Power of a Power Property and applying the Power of a Product Property for rational exponents using an example that involves both properties.

Possible answer: Consider the expression $\left(4^{\frac{2}{3}} x^{\frac{4}{3}}\right)^{\frac{3}{4}}$. This is the $\frac{3}{4}$ power of the product of $4^{\frac{2}{3}}$ and $x^{\frac{4}{3}}$. The result is the product of the $\frac{3}{4}$ power of each factor: $\left(4^{\frac{2}{3}}\right)^{\frac{3}{4}} \left(x^{\frac{4}{3}}\right)^{\frac{3}{4}}$. This expression contains the $\frac{3}{4}$ power of the power $4^{\frac{2}{3}}$ and the $\frac{3}{4}$ power of the power $x^{\frac{4}{3}}$. Simplifying then gives $\left(4^{\frac{2}{3}}\right)^{\frac{3}{4}} \left(x^{\frac{4}{3}}\right)^{\frac{3}{4}} = 4^{\frac{1}{2}} \cdot x^1 = 2x$

Can you use the properties of rational exponents to simplify $\sqrt{a} \cdot \sqrt[3]{b}$? Explain. **No; in rational exponent form, the expression is $a^{\frac{1}{2}} \cdot b^{\frac{1}{3}}$. Because the bases are different, the product of powers property does not apply, and the expression cannot be simplified.**

SUMMARIZE THE LESSON

How can the properties of exponents be applied to the simplification of expressions containing rational exponents and to those containing radicals? **For expressions containing rational exponents, the properties of exponents can be applied directly. For radical expressions, convert the radical expressions to exponent form using the fact that $\sqrt[n]{a^m} = a^{\frac{m}{n}}$, and then apply the properties.**