59. Is it possible to define $f(x) = \dfrac{1}{x-9}$ at $x = 9$ so that the function becomes continuous at 9? How does this discontinuity differ from the discontinuity in Exercise 58?

60. For the function

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ Ax - 3 & \text{if } x \ge 1 \end{cases}$$

find A so that the function is continuous at 1.

Group Exercise

61. In this exercise, the group will define three piecewise functions. Each function should have three pieces and two values of x at which the pieces change.

a. Define and graph a piecewise function that is continuous at both values of x where the pieces change.

b. Define and graph a piecewise function that is continuous at one value of x where the pieces change and discontinuous at the other value of x where the pieces change.

c. Define and graph a piecewise function that is discontinuous at both values of x where the pieces change.

At the end of the activity, group members should turn in the functions and their graphs. Do not use any of the piecewise functions or graphs that appear anywhere in this book.

Preview Exercises

Exercises 62–64 will help you prepare for the material covered in the next section. In each exercise, find the indicated difference quotient and simplify.

62. If $f(x) = x^2 + x$, find $\dfrac{f(2+h) - f(2)}{h}$.

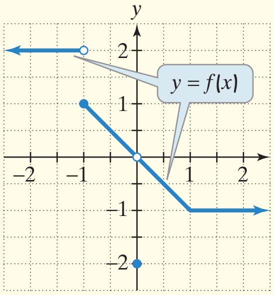
63. If $f(x) = x^3$, find $\dfrac{f(x+h) - f(x)}{h}$.

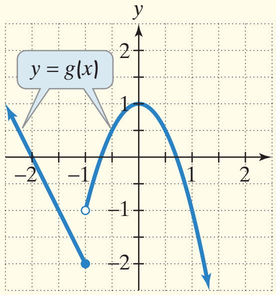
64. If $s(t) = -16t^2 + 48t + 160$, find $\dfrac{s(a+h) - s(a)}{h}$.

Chapter 11 Mid-Chapter Check Point

What you know: We learned that $\lim\limits_{x \to a} f(x) = L$ means that as x gets closer to a, but remains unequal to a, the corresponding values of $f(x)$ get closer to L. We found limits using tables, graphs, and properties of limits. The quotient property for limits did not apply to fractional expressions in which the limit of the denominator is zero. In these cases, rewriting the expression using factoring or rationalizing the numerator or denominator was helpful before finding the limit. We saw that if the left-hand limit, $\lim\limits_{x \to a^-} f(x)$, ($x$ approaches a from the left) is not equal to the right-hand limit, $\lim\limits_{x \to a^+} f(x)$, ($x$ approaches a from the right), then $\lim\limits_{x \to a} f(x)$ does not exist. Finally, we defined continuity in terms of limits. A function f is continuous at a when f is defined at a, $\lim\limits_{x \to a} f(x)$ exists, and $\lim\limits_{x \to a} f(x) = f(a)$. If f is not continuous at a, we say that f is discontinuous at a.

In Exercises 1–7, use the graphs of f and g to find the indicated limit or function value, or state that the limit or function value does not exist.





1. $\lim\limits_{x \to -1^-} f(x)$

2. $\lim\limits_{x \to -1^+} f(x)$

3. $\lim\limits_{x \to -1} f(x)$

4. $\lim\limits_{x \to 1} [f(x) + g(x)]$

5. $\lim\limits_{x \to 0} [f(x) - g(x)]$

6. $(f - g)(0)$

7. $\lim\limits_{x \to 1} \sqrt{10 + f(x)}$

8. Use the graph of f shown above to determine for what numbers the function is discontinuous. Then use the definition of continuity to verify each discontinuity.

In Exercises 9–11, use the table to find the indicated limit.

x	−0.03	−0.02	−0.01	−0.007	0.007	0.01	0.02	0.03
$f(x) = \dfrac{\sin x}{2x^2 - x}$	−0.9433	−0.9615	−0.9804	−0.9862	−1.014	−1.02	−1.042	−1.064
$g(x) = \dfrac{e^x - \tan x}{\cos^2 x}$	1.0014	1.0006	1.0002	1.0001	1.0001	1.0001	1.0006	1.0013

(In Exercises 9–11, be sure to refer to the table at the bottom of the previous page.)

9. $\lim_{x\to 0} f(x)$ **10.** $\lim_{x\to 0} g(x)$ **11.** $\lim_{x\to 0} \dfrac{4g(x)}{[f(x)]^2}$

In Exercises 12–17, find the limits.

12. $\lim_{x\to -2}(x^3 - x + 5)$

13. $\lim_{x\to 3}\sqrt{x^2 - 3x + 4}$

14. $\lim_{x\to 5}\dfrac{2x^2 - x + 4}{x - 1}$

15. $\lim_{x\to 5}\dfrac{2x^2 - 7x - 15}{x - 5}$

16. $\lim_{x\to 0}\dfrac{\sqrt{x^2+9} - 3}{x^2}$

17. $\lim_{x\to 0}\dfrac{\dfrac{1}{x+10} - \dfrac{1}{x}}{x}$

In Exercises 18–19, a piecewise function is given. Use the function to find the indicated limit, or state that the limit does not exist.

18. $f(x) = \begin{cases} 9 - 2x & \text{if } x < 4 \\ \sqrt{x - 4} & \text{if } x \ge 4 \end{cases}$

a. $\lim_{x\to 4^-} f(x)$ **b.** $\lim_{x\to 4^+} f(x)$ **c.** $\lim_{x\to 4} f(x)$

19. $f(x) = \begin{cases} \dfrac{x^4 - 16}{x - 2} & \text{if } x \ne 2 \\ 32 & \text{if } x = 2 \end{cases}$

a. $\lim_{x\to 2^-} f(x)$ **b.** $\lim_{x\to 2^+} f(x)$ **c.** $\lim_{x\to 2} f(x)$

In Exercises 20–21, use the definition of continuity to determine whether f is continuous at a.

20. $f(x) = \begin{cases} \sqrt{3 - x} & \text{if } x \le 3 \\ x^2 - 3x & \text{if } x > 3 \end{cases}$

$a = 3$

21. $f(x) = \begin{cases} \dfrac{(x+3)^2 - 9}{x} & \text{if } x \ne 0 \\ 6 & \text{if } x = 0 \end{cases}$

$a = 0$

22 Determine for what numbers, if any, the following function is discontinuous:

$$f(x) = \begin{cases} \dfrac{x^2 - 1}{x + 1} & \text{if } x < -1 \\ 2x & \text{if } -1 \le x \le 5. \\ 3x - 4 & \text{if } x > 5 \end{cases}$$

Section 11.4 Introduction to Derivatives

Objectives

1. Find slopes and equations of tangent lines.
2. Find the derivative of a function.
3. Find average and instantaneous rates of change.
4. Find instantaneous velocity.

Things change over time and most changes occur at uneven rates. This is illustrated in the chapter opener (page 1037) with a sequence of photos of a young boy transforming into an adult. What does calculus have to say about this radical transformation?

In this section, we will see how calculus allows motion and change to be analyzed by "freezing the frame" of a continuously changing process, instant by instant. For example, **Figure 11.15** shows a male's changing height over intervals of time. Over the period of time from P to D, his average rate of growth is his change in height—that is, his height at time D minus his height at time P—divided by the change in time from P to D.

The lines PD, PC, PB, and PA shown in **Figure 11.15** have slopes that show the man's average growth rates for successively shorter periods of time. Calculus makes these time frames so small that their limit approaches a single point—that is, a single instant in time. This point is shown as point P in **Figure 11.15**. The slope of the line that touches the graph at P gives the man's growth rate at one instant in time, P.

Keep this informal discussion of this man and his growth rate in mind as you read this section. We begin with the calculus that describes the slope of the line that touches the graph in **Figure 11.15** at P.

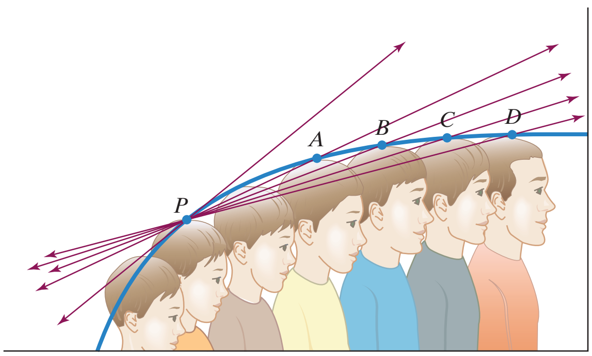


Figure 11.15