

Chapter 11 Summary, Review, and Test

Summary

DEFINITIONS AND CONCEPTS

EXAMPLES

11.1 Finding Limits Using Tables and Graphs

a. Limit Notation and Its Description

$\lim_{x \rightarrow a} f(x) = L$ is read “the limit of $f(x)$ as x approaches a equals the number L .” This means that as x gets closer to a , but remains unequal to a , the corresponding values of $f(x)$ get closer to L .

b. Limits can be found using tables.

Ex. 1, p. 1040;
Ex. 2, p. 1040

c. Limits can be found using graphs.

Ex. 3, p. 1042;
Ex. 4, p. 1042

d. Left-Hand Limit

$\lim_{x \rightarrow a^-} f(x) = L$ is read “the limit of $f(x)$ as x approaches a from the left equals L .” This means that as x gets closer to a , but remains less than a , the corresponding values of $f(x)$ get closer to L .

e. Right-Hand Limit

$\lim_{x \rightarrow a^+} f(x) = L$ is read “the limit of $f(x)$ as x approaches a from the right equals L .” This means that as x gets closer to a , but remains greater than a , the corresponding values of $f(x)$ get closer to L .

f. If $\lim_{x \rightarrow a^-} f(x) \neq \lim_{x \rightarrow a^+} f(x)$, then $\lim_{x \rightarrow a} f(x)$ does not exist.

Ex. 5, p. 1044

11.2 Finding Limits Using Properties of Limits

a. Properties of limits are given in the box on page 1057.

Ex. 1–Ex. 11,
pp. 1050–1057

b. Properties of limits can be used to find one-sided limits.

Ex. 12, p. 1058

c. When taking the limit of a fractional expression in which the limit of the denominator is zero, the quotient property for limits cannot be used. Rewriting the expression using factoring or rationalizing the numerator or denominator may be helpful before the limit is found.

Ex. 13, p. 1058;
Ex. 14, p. 1059

11.3 Limits and Continuity

a. A function f is continuous at a when f is defined at a , $\lim_{x \rightarrow a} f(x)$ exists, and $\lim_{x \rightarrow a} f(x) = f(a)$. If f is not continuous at a , we say that f is discontinuous at a .

Ex. 1, p. 1063;
Ex. 2, p. 1065

11.4 Introduction to Derivatives

a. The slope of the tangent line to the graph of a function $y = f(x)$ at $(a, f(a))$ is given by

Ex. 1, p. 1073;
Ex. 2, p. 1074

$$m_{\text{tan}} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

provided that this limit exists. The limit also describes the slope of the graph of f at $(a, f(a))$.

b. The Derivative of a Function

The derivative of f at x is given by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

provided that this limit exists. The derivative gives the slope of f for any value at x in the domain of f .

Ex. 3, p. 1076

c. The derivative of f at a , $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$, gives the instantaneous rate of change of f with respect to x at a . Expressions for average and instantaneous rates of change are given in the box on page 1077.

Ex. 4, p. 1077

d. If a function expresses an object's position, $s(t)$, in terms of time, t , the instantaneous velocity of the object at time $t = a$ is

Ex. 5, p. 1079

$$s'(a) = \lim_{h \rightarrow 0} \frac{s(a+h) - s(a)}{h}$$

Review Exercises

11.1

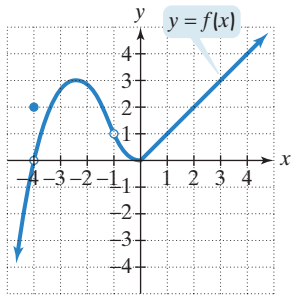
In Exercises 1–3, construct a table to find the indicated limit.

1. $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$

2. $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$

3. $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$

In Exercises 4–8, use the graph of f to find the indicated limit or function value.



4. $\lim_{x \rightarrow -4} f(x)$

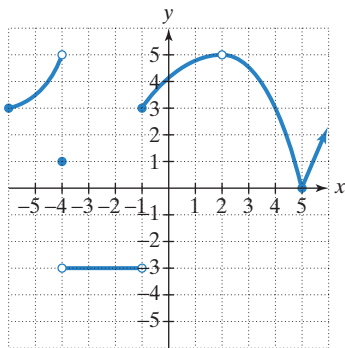
5. $\lim_{x \rightarrow -1} f(x)$

6. $\lim_{x \rightarrow 3} f(x)$

7. $f(-4)$

8. $f(3)$

In Exercises 9–23, use the graph of function f to find the indicated limit or function value, or state that the limit or function value does not exist.



9. $\lim_{x \rightarrow -6^+} f(x)$

10. $\lim_{x \rightarrow -4^-} f(x)$

11. $\lim_{x \rightarrow -4^+} f(x)$

12. $\lim_{x \rightarrow -4} f(x)$

13. $f(-4)$

14. $\lim_{x \rightarrow -1^+} f(x)$

15. $\lim_{x \rightarrow -1^-} f(x)$

16. $\lim_{x \rightarrow -1} f(x)$

17. $f(-1)$

18. $f(2)$

19. $\lim_{x \rightarrow 2^-} f(x)$

20. $\lim_{x \rightarrow 2^+} f(x)$

21. $\lim_{x \rightarrow 2} f(x)$

22. $\lim_{x \rightarrow 5} f(x)$

23. $f(5)$

In Exercises 24–26, graph each function. Then use your graph to find the indicated limit.

24. $f(x) = \frac{x^2 - 9}{x - 3}$, $\lim_{x \rightarrow 3} f(x)$

25. $f(x) = \sin x$, $\lim_{x \rightarrow \frac{3\pi}{2}} f(x)$

26. $f(x) = \begin{cases} 1 - x & \text{if } x < 0 \\ \cos x & \text{if } x \geq 0 \end{cases}$, $\lim_{x \rightarrow 0} f(x)$

11.2

In Exercises 27–37, find the limit.

27. $\lim_{x \rightarrow 4} (2x^2 - 5x + 3)$

28. $\lim_{x \rightarrow -1} (-2x^3 - x + 5)$

29. $\lim_{x \rightarrow -3} (x^2 + 1)^3$

31. $\lim_{x \rightarrow 5} \frac{11x - 3}{x^2 + 1}$

33. $\lim_{x \rightarrow 7} \frac{5x - 35}{x - 7}$

35. $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x^2 + x}$

37. $\lim_{x \rightarrow 0} \frac{1}{\frac{1}{x+5} - \frac{1}{5}}$

30. $\lim_{x \rightarrow 4} \sqrt{x^2 + 9}$

32. $\lim_{x \rightarrow -4} \frac{x^2 - 16}{x + 4}$

34. $\lim_{x \rightarrow 0} \frac{\sqrt{x+100} - 10}{x}$

36. $\lim_{x \rightarrow 100} \frac{\sqrt{x} - 10}{x - 100}$

In Exercises 38–40, a piecewise function is given. Use properties of limits to find the indicated limit, or state that the limit does not exist.

38. $f(x) = \begin{cases} x^2 + 1 & \text{if } x < 2 \\ 3x + 1 & \text{if } x \geq 2 \end{cases}$

a. $\lim_{x \rightarrow 2^-} f(x)$

b. $\lim_{x \rightarrow 2^+} f(x)$

c. $\lim_{x \rightarrow 2} f(x)$

39. $f(x) = \begin{cases} \sqrt[3]{x^2 + 7} & \text{if } x < 1 \\ 4x & \text{if } x \geq 1 \end{cases}$

a. $\lim_{x \rightarrow 1^-} f(x)$

b. $\lim_{x \rightarrow 1^+} f(x)$

c. $\lim_{x \rightarrow 1} f(x)$

40. $f(x) = \begin{cases} \frac{x^2 - 25}{x + 5} & \text{if } x \neq -5 \\ 13 & \text{if } x = -5 \end{cases}$

a. $\lim_{x \rightarrow -5^-} f(x)$

b. $\lim_{x \rightarrow -5^+} f(x)$

c. $\lim_{x \rightarrow -5} f(x)$

11.3

In Exercises 41–45, use the definition of continuity to determine whether f is continuous at a .

41. $f(x) = 3x^2 - 2x + 1$
 $a = 4$

42. $f(x) = \frac{x^2 - 9}{x + 3}$
 $a = -3$

43. $f(x) = \begin{cases} \frac{x^2 + 5x}{x^2 - 5x} & \text{if } x \neq 0 \\ -2 & \text{if } x = 0 \end{cases}$
 $a = 0$

44. $f(x) = \begin{cases} \frac{x^2 + x}{x^2 - 3x - 4} & \text{if } x \neq -1 \\ \frac{1}{5} & \text{if } x = -1 \end{cases}$
 $a = -1$

45. $f(x) = \begin{cases} 3x & \text{if } x < 2 \\ 5 & \text{if } x = 2 \\ x + 4 & \text{if } x > 2 \end{cases}$
 $a = 2$

In Exercises 46–51, determine for what numbers, if any, the given function is discontinuous.

46. $f(x) = x^3 + 5x^2 - 1$

47. $f(x) = \frac{x - 1}{(x - 1)(x + 3)}$

48. $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$

49. $f(x) = \begin{cases} 4x & \text{if } x < 5 \\ x^2 - 5 & \text{if } x \geq 5 \end{cases}$

$$50. f(x) = \begin{cases} \frac{x^2 - 4}{x + 2} & \text{if } x \neq -2 \\ 4 & \text{if } x = -2 \end{cases}$$

$$51. f(x) = \begin{cases} \frac{x^2 - 121}{x - 11} & \text{if } x \neq 11 \\ 20 & \text{if } x = 11 \end{cases}$$

11.4

In Exercises 52–53,

- a. Find the slope of the tangent line to the graph of f at the given point.
- b. Find the slope-intercept equation of the tangent line to the graph of f at the given point.

52. $f(x) = 2x^2 + 5x$ at $(1, 7)$

53. $f(x) = x^2 - 7x - 4$ at $(-1, 4)$

In Exercises 54–57,

- a. Find $f'(x)$.
- b. Find the slope of the tangent line to the graph of f at each of the two values of x given to the right of the function.

54. $f(x) = 3x^2 + 12x - 1$; $x = -2, x = 1$

55. $f(x) = 2x^3 - x$; $x = -1, x = 1$

56. $f(x) = \frac{1}{x}$; $x = -2, x = 2$ 57. $f(x) = \sqrt{x}$; $x = 36, x = 81$

58. The function $f(x) = 5x^2$ describes the volume of a rectangular box, $f(x)$, in cubic inches, whose square base has sides that each measure x inches and whose height is 5 inches. If x is changing,

- a. Find the average rate of change of the volume with respect to x as x changes from 2 inches to 2.1 inches and from 2 inches to 2.01 inches.
- b. Find the instantaneous rate of change of the volume with respect to x at the moment when $x = 2$ inches.

59. The function $f(x) = \frac{4}{3}\pi x^3$ describes the volume, $f(x)$, of a sphere of radius x inches. If the radius is changing, find the instantaneous rate of change of the volume with respect to the radius when the radius is 5 inches. Express the answer in terms of π .

60. A baseball is thrown straight upward from a height of 5 feet with an initial velocity of 80 feet per second. The function

$$s(t) = -16t^2 + 80t + 5$$

describes the ball's height above the ground, $s(t)$, in feet, t seconds after it is thrown.

- a. What is the instantaneous velocity of the ball 2 seconds after it is thrown? 4 seconds after it is thrown?
- b. The ball reaches its maximum height above the ground when the instantaneous velocity is zero. After how many seconds does the ball reach its maximum height? What is the maximum height?

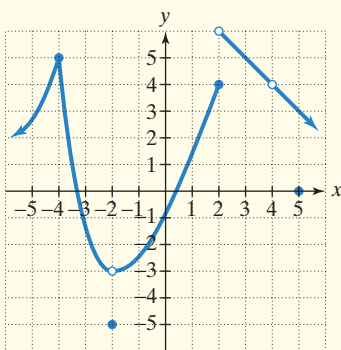


CHAPTER
Test Prep
VIDEOS

Chapter 11 Test

1. Construct a table to find $\lim_{x \rightarrow 9} \frac{9 - x}{3 - \sqrt{x}}$.

In Exercises 2–7, use the graph of function f to find the indicated limit or function value, or state that the limit or function value does not exist.



- 2. $\lim_{x \rightarrow -2} f(x)$
- 3. $f(-2)$
- 4. $\lim_{x \rightarrow 2^-} f(x)$
- 5. $\lim_{x \rightarrow 2^+} f(x)$
- 6. $\lim_{x \rightarrow 2} f(x)$
- 7. $\lim_{x \rightarrow 4} f(x)$

In Exercises 8–10, find the limit.

8. $\lim_{x \rightarrow -2} (x^2 + x + 1)^4$

9. $\lim_{x \rightarrow -1} \frac{x^2 - x - 2}{x + 1}$

10. $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$

In Exercises 11–12, determine whether f is continuous at a .

11. $f(x) = \begin{cases} \frac{x^2 - 1}{x + 1} & \text{if } x \neq -1 \\ 6 & \text{if } x = -1 \end{cases}$

$a = -1$

12. $f(x) = \begin{cases} 2 - x & \text{if } x \leq 2 \\ x^2 - 2x & \text{if } x > 2 \end{cases}$

$a = 2$

In Exercises 13–14, find $f'(x)$.

13. $f(x) = x^2 - 5x + 1$

14. $f(x) = \frac{10}{x}$

15. Find the slope-intercept equation of the tangent line to the graph of $f(x) = x^2$ at $(-3, 9)$.

16. A ball is thrown straight upward. The function

$$s(t) = -16t^2 + 72t$$

describes the ball's height above the ground, $s(t)$, in feet, t seconds after it is thrown. What is the instantaneous velocity of the ball 3 seconds after it is thrown?

Cumulative Review Exercises (Chapters P–I I)

Solve each equation or inequality in Exercises 1–5.

- $\frac{1}{x+2} > \frac{3}{x+1}$
- $2x^3 + 11x^2 - 7x - 6 = 0$
- $|2x + 4| > 3$
- $\cos^2 x + \sin x + 1 = 0, 0 \leq x < 2\pi$
- $\log_4(x^2 - 9) - \log_4(x + 3) = 3$

In Exercises 6–15, graph each equation, function, or system in a rectangular coordinate system.

- $f(x) = x^3 + x^2 - 12x$
- $f(x) = \frac{2x^2 - 5x + 2}{x^2 - 4}$
- $f(x) = \begin{cases} -x + 1 & \text{if } -1 \leq x < 1 \\ 2 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$
- $y = 2 \sin\left(2x + \frac{\pi}{2}\right)$ (Graph one period.)
- $y = \frac{1}{2} \sec 2\pi x, 0 \leq x \leq 2$
- $\begin{cases} x - 2y \leq 4 \\ x \geq 2 \end{cases}$
- $x^2 - 4y^2 - 4x - 24y - 48 = 0$
- $f(x) = \sqrt{x}, g(x) = \sqrt{x-2} + 1$ (Graph f and g in the same rectangular coordinate system.)
- $x = 3 \sin t, y = 4 \cos t + 2; 0 \leq t \leq 2\pi$
- $2x^2 + 5xy + 2y^2 - \frac{9}{2} = 0$
- Find $f'(x)$ if $f(x) = -2x^2 + 7x - 1$.
- Find $f^{-1}(x)$ if $f(x) = 7x - 1$.
- Find the limit: $\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x^2 + 2x - 3}$.
- Expand and simplify: $(x^2 - 3y)^4$.
- Write the slope-intercept form of the equation of the line passing through the point $(2, -3)$ and parallel to the line whose equation is $2x + y - 6 = 0$.
- Find the dot product $\mathbf{v} \cdot \mathbf{w}$ and the angle between \mathbf{v} and \mathbf{w} :

$$\mathbf{v} = -2\mathbf{i} + \mathbf{j}, \quad \mathbf{w} = 4\mathbf{i} - 3\mathbf{j}.$$

- Find the partial fraction decomposition for

$$\frac{1}{x(x^2 + x + 1)}.$$

Verify each identity in Exercises 23–24.

- $\tan \theta + \cot \theta = \sec \theta \csc \theta$
- $\tan(\theta + \pi) = \tan \theta$

- If $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 2 & 1 \end{bmatrix}$, find BA .

- Graph the polar equation: $r = 4 \sin \theta$.
- Express $h(x) = (x^2 - 3x + 7)^9$ as a composition of two functions f and g such that $h(x) = (f \circ g)(x)$.

- Solve using matrices:

$$\begin{cases} 2x - y - 2z = -1 \\ x - 2y - z = 1 \\ x + y + z = 1. \end{cases}$$

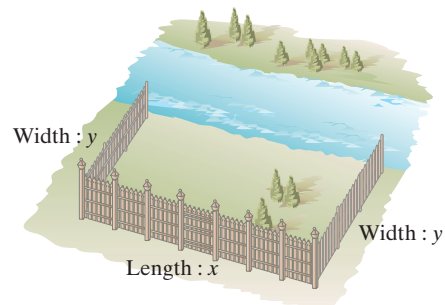
- Use the formula for the sum of the first n terms of a geometric sequence to find $\sum_{i=1}^6 4(-2)^i$.

- Use DeMoivre's Theorem to find

$$\left[\sqrt{2}(\cos 15^\circ + i \sin 15^\circ)\right]^4.$$

Write the answer in rectangular form.

- A bank loaned out \$120,000, part of it at 8% per year and the rest at 18% per year.
 - Express the interest, I , on the two loans as a function of the amount loaned at 8%, x .
 - If the interest received totaled \$10,000, how much was loaned at each rate?
- A machine produces open boxes using square sheets of metal. The machine cuts equal-sized squares measuring 9 centimeters on a side from each corner. Then the machine shapes the metal into an open box by turning up the sides. If each box must have a volume of 225 cubic centimeters, what should be the dimensions of the piece of sheet metal?
- You have 200 feet of fencing to enclose a small rectangular garden with one side against a barn. If you do not fence the side along the barn, find the length and width of the garden that will maximize its area. What is the largest area that can be enclosed?
- Use Newton's Law of Cooling, $T = C + (T_0 - C)e^{kt}$, to solve this exercise. You remove a pie that has a temperature of 375°F from the oven. You leave the pie in a room whose temperature is 72°F. After 60 minutes, the temperature of the pie is 75°F.
 - Write a model for the temperature of the pie, T , after t minutes.
 - When will the temperature of the pie be 250°F?
- You just purchased a rectangular waterfront lot along a river's edge. The area of the lot is 60,000 square feet. To create a sense of privacy, you decide to fence along three sides, excluding the side that fronts the river. An expensive fencing along the lot's front length costs \$25 per foot. An inexpensive fencing along the two side widths costs only \$5 per foot. Express the total cost, C , of fencing along the three sides as a function of the lot's length, x .

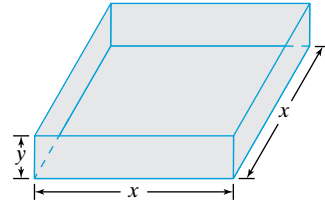


36. Two ships leave a harbor at the same time. One ship travels at a bearing of $N42^\circ E$ for 23 miles. The other ship travels at a bearing of $N38^\circ W$ for 72 miles. After both ships are anchored, how far apart are they? Round to the nearest tenth of a mile.
37. At a fixed temperature, the volume of a given mass of gas varies inversely as the pressure applied to the gas. A certain mass of gas has a volume of 40 cubic inches when the pressure is 22 pounds. What is the volume of the gas when the pressure is 30 pounds?
38. A ball is thrown straight upward. The function

$$s(t) = -16t^2 + 40t$$

describes the ball's height above the ground, $s(t)$, in feet, t seconds after it is thrown. What is the instantaneous velocity of the ball 2 seconds after it is thrown?

39. The figure shows an open box with a square base. The box is to have a volume of 4 cubic feet. Express the surface area of the box, A , as a function of the length of a side of its square base, x .



40. The function $f(x) = -2.32x^2 + 76.58x - 559.87$ models the percentage of U.S. students, $f(x)$, who are x years old who say their school is not drug free, where $12 \leq x \leq 17$. At what age do 70% of U.S. students say that their school is not drug free? Round to the nearest tenth of a year.