38. Let $C(x) = \begin{cases} 30 & \text{if} \quad 0 \le t \le 200 \\ 30 + 0.40(t - 200) & \text{if} \quad t > 200 \end{cases}$.

a. Find $C(150)$. **b.** Find $C(250)$.

In Exercises 39–42, write a linear function in slope-intercept form whose graph satisfies the given conditions.

39. Slope $= -2$, passing through $(-4, 3)$

40. Passing through $(-1, -5)$ and $(2, 1)$

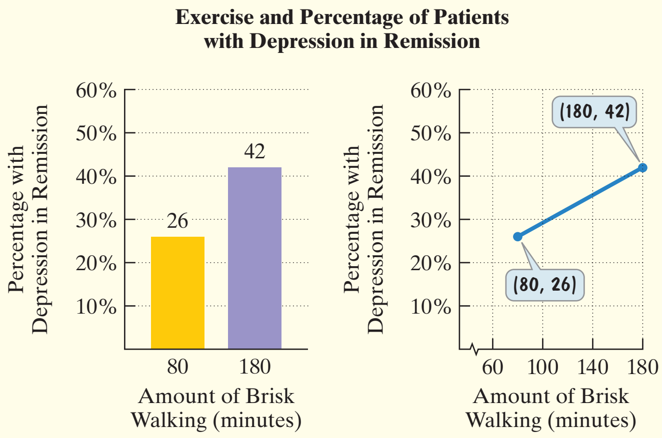
41. Passing through $(3, -4)$ and parallel to the line whose equation is $3x - y - 5 = 0$

42. Passing through $(-4, -3)$ and perpendicular to the line whose equation is $2x - 5y - 10 = 0$

43. Determine whether the line through $(2, -4)$ and $(7, 0)$ is parallel to a second line through $(-4, 2)$ and $(1, 6)$.

44. Exercise is useful not only in preventing depression, but also as a treatment. The graphs in the next column show the percentage of patients with depression in remission when exercise (brisk walking) was used as a treatment. (The control group that engaged in no exercise had 11% of the patients in remission.)

a. Find the slope of the line passing through the two points shown by the voice balloons. Express the slope as a decimal.



Source: Newsweek, March 26, 2007

b. Use your answer from part (a) to complete this statement:

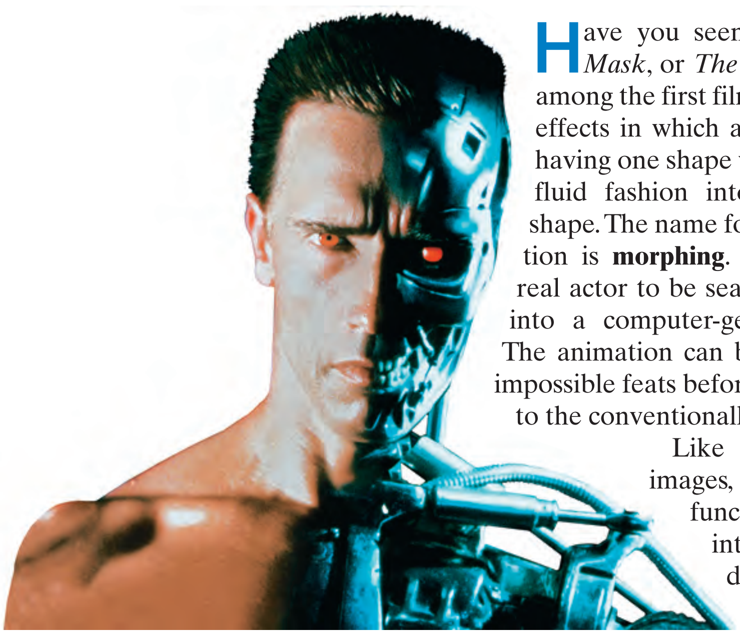
For each minute of brisk walking, the percentage of patients with depression in remission increased by ______ %. The rate of change is ______ % per ______.

45. Find the average rate of change of $f(x) = 3x^2 - x$ from $x_1 = -1$ to $x_2 = 2$.

Section 1.6 Transformations of Functions

Objectives

1. Recognize graphs of common functions.
2. Use vertical shifts to graph functions.
3. Use horizontal shifts to graph functions.
4. Use reflections to graph functions.
5. Use vertical stretching and shrinking to graph functions.
6. Use horizontal stretching and shrinking to graph functions.
7. Graph functions involving a sequence of transformations.



Have you seen *Terminator 2, The Mask*, or *The Matrix?* These were among the first films to use spectacular effects in which a character or object having one shape was transformed in a fluid fashion into a quite different shape. The name for such a transformation is **morphing**. The effect allows a real actor to be seamlessly transformed into a computer-generated animation. The animation can be made to perform impossible feats before it is morphed back to the conventionally filmed image.

Like transformed movie images, the graph of one function can be turned into the graph of a different function. To do this, we need to rely on a function's equation. Knowing that a graph is a transformation of a familiar graph makes graphing easier.

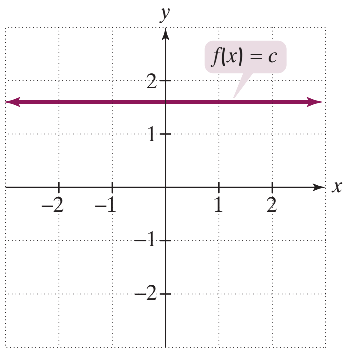
1 Recognize graphs of common functions.

Graphs of Common Functions

Table 1.3 on the next page gives names to seven frequently encountered functions in algebra. The table shows each function's graph and lists characteristics of the function. Study the shape of each graph and take a few minutes to verify the function's characteristics from its graph. Knowing these graphs is essential for analyzing their transformations into more complicated graphs.

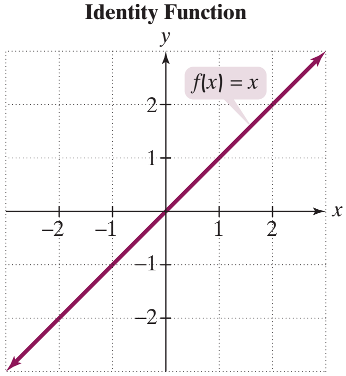
Table 1.3 Algebra's Common Graphs

Constant Function



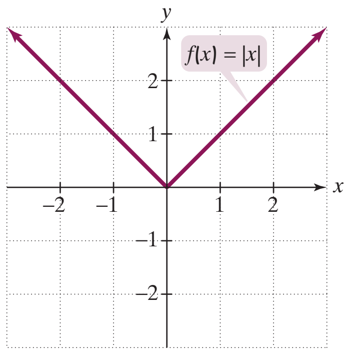
- Domain: $(-\infty, \infty)$
- Range: the single number c
- Constant on $(-\infty, \infty)$
- Even function

Identity Function



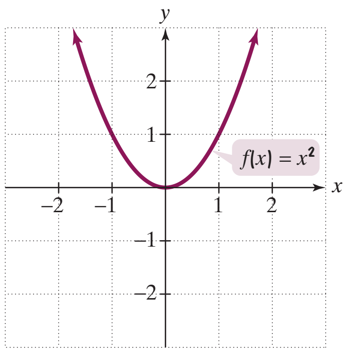
- Domain: $(-\infty, \infty)$
- Range: $(-\infty, \infty)$
- Increasing on $(-\infty, \infty)$
- Odd function

Absolute Value Function



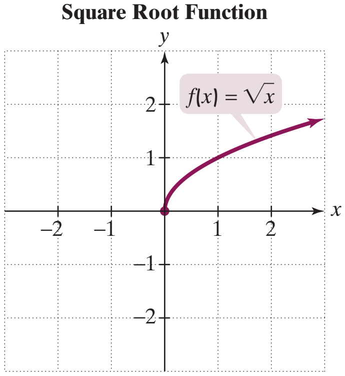
- Domain: $(-\infty, \infty)$
- Range: $[0, \infty)$
- Decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$
- Even function

Standard Quadratic Function



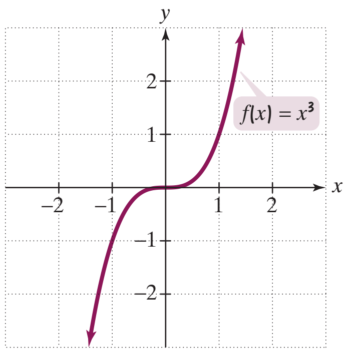
- Domain: $(-\infty, \infty)$
- Range: $[0, \infty)$
- Decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$
- Even function

Square Root Function



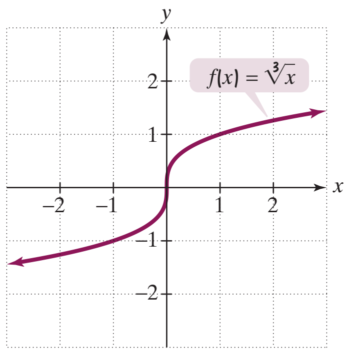
- Domain: $[0, \infty)$
- Range: $[0, \infty)$
- Increasing on $(0, \infty)$
- Neither even nor odd

Standard Cubic Function



- Domain: $(-\infty, \infty)$
- Range: $(-\infty, \infty)$
- Increasing on $(-\infty, \infty)$
- Odd function

Cube Root Function



- Domain: $(-\infty, \infty)$
- Range: $(-\infty, \infty)$
- Increasing on $(-\infty, \infty)$
- Odd function

Discovery

The study of how changing a function's equation can affect its graph can be explored with a graphing utility. Use your graphing utility to verify the hand-drawn graphs as you read this section.

2 Use vertical shifts to graph functions.

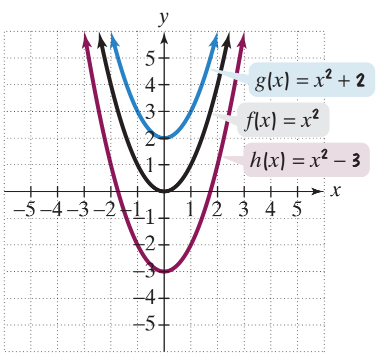


Figure 1.55 Vertical shifts

Vertical Shifts

Let's begin by looking at three graphs whose shapes are the same. **Figure 1.55** shows the graphs. The black graph in the middle is the standard quadratic function, $f(x) = x^2$. Now, look at the blue graph on the top. The equation of this graph, $g(x) = x^2 + 2$, adds 2 to the right side of $f(x) = x^2$. The y-coordinate of each point of g is 2 more than the corresponding y-coordinate of each point of f. What effect does this have on the graph of f? It shifts the graph vertically up by 2 units.

$$g(x) = x^2 + 2 = f(x) + 2$$

The graph of g | shifts the graph of f up 2 units.

Finally, look at the red graph on the bottom in **Figure 1.55**. The equation of this graph, $h(x) = x^2 - 3$, subtracts 3 from the right side of $f(x) = x^2$. The y-coordinate of each point of h is 3 less than the corresponding y-coordinate of

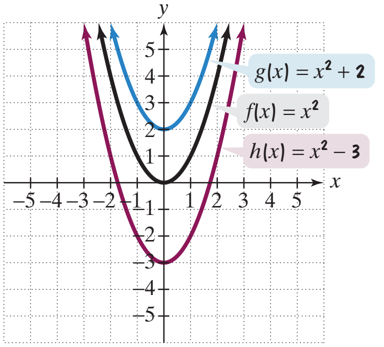


Figure 1.55 (repeated) Vertical shifts

each point of f. What effect does this have on the graph of f? It shifts the graph vertically down by 3 units.

$$h(x) = x^2 - 3 = f(x) - 3$$

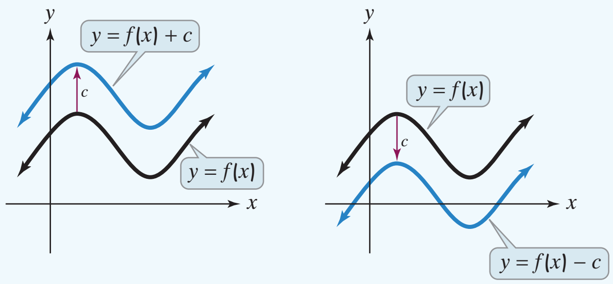
The graph of h — shifts the graph of f down 3 units.

In general, if c is positive, $y = f(x) + c$ shifts the graph of f upward c units and $y = f(x) - c$ shifts the graph of f downward c units. These are called **vertical shifts** of the graph of f.

Vertical Shifts

Let f be a function and c a positive real number.

- The graph of $y = f(x) + c$ is the graph of $y = f(x)$ shifted c units vertically upward.
- The graph of $y = f(x) - c$ is the graph of $y = f(x)$ shifted c units vertically downward.



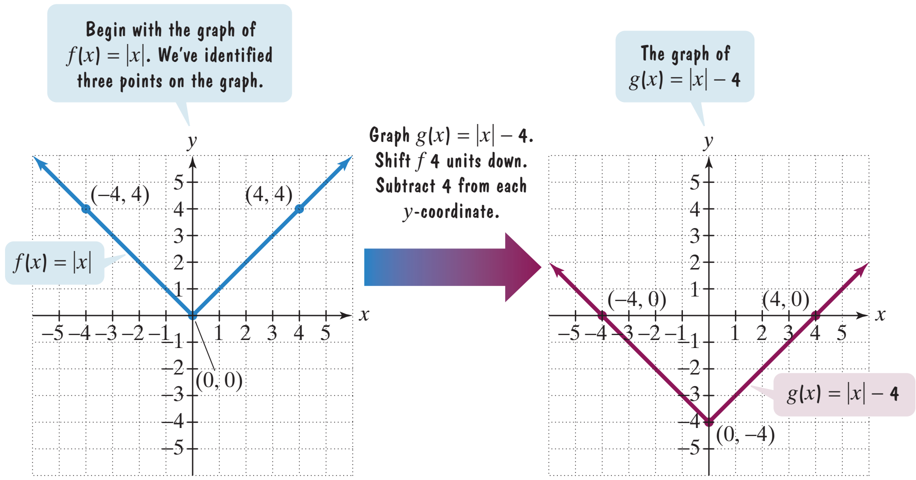
Study Tip

To keep track of transformations, identify a number of points on the given function's graph. Then analyze what happens to the coordinates of these points with each transformation.

EXAMPLE 1 Vertical Shift Downward

Use the graph of $f(x) = |x|$ to obtain the graph of $g(x) = |x| - 4$.

Solution The graph of $g(x) = |x| - 4$ has the same shape as the graph of $f(x) = |x|$. However, it is shifted down vertically 4 units.



Check Point 1 Use the graph of $f(x) = |x|$ to obtain the graph of $g(x) = |x| + 3$.

3 Use horizontal shifts to graph functions.

Horizontal Shifts

We return to the graph of $f(x) = x^2$, the standard quadratic function. In **Figure 1.56**, the graph of function f is in the middle of the three graphs. By contrast to the vertical shift situation, this time there are graphs to the left and to the right of the graph of f. Look at the blue graph on the right. The equation of this graph, $g(x) = (x - 3)^2$, subtracts 3 from each value of x before squaring it. What effect does this have on the graph of $f(x) = x^2$? It shifts the graph horizontally to the right by 3 units.

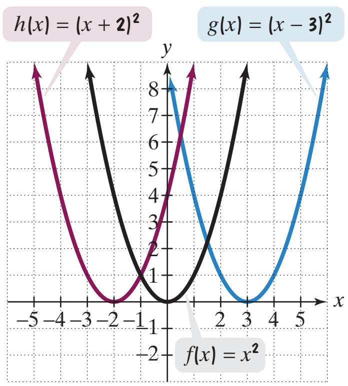


Figure 1.56 Horizontal shifts

$$g(x) = (x - 3)^2 = f(x - 3)$$

The graph of g shifts the graph of f 3 units to the right.

Does it seem strange that *subtracting* 3 in the domain causes a shift of 3 units to the *right*? Perhaps a partial table of coordinates for each function will numerically convince you of this shift.

x	$f(x) = x^2$
−2	$(-2)^2 = 4$
−1	$(-1)^2 = 1$
0	$0^2 = 0$
1	$1^2 = 1$
2	$2^2 = 4$

x	$g(x) = (x - 3)^2$
1	$(1 - 3)^2 = (-2)^2 = 4$
2	$(2 - 3)^2 = (-1)^2 = 1$
3	$(3 - 3)^2 = 0^2 = 0$
4	$(4 - 3)^2 = 1^2 = 1$
5	$(5 - 3)^2 = 2^2 = 4$

Notice that for the values of $f(x)$ and $g(x)$ to be the same, the values of x used in graphing g must each be 3 units greater than those used to graph f. For this reason, the graph of g is the graph of f shifted 3 units to the right.

Now, look at the red graph on the left in **Figure 1.56**. The equation of this graph, $h(x) = (x + 2)^2$, adds 2 to each value of x before squaring it. What effect does this have on the graph of $f(x) = x^2$? It shifts the graph horizontally to the left by 2 units.

$$h(x) = (x + 2)^2 = f(x + 2)$$

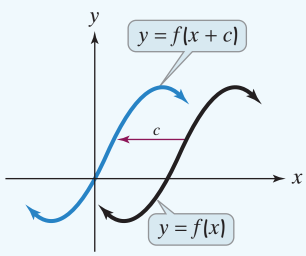
The graph of h shifts the graph of f 2 units to the left.

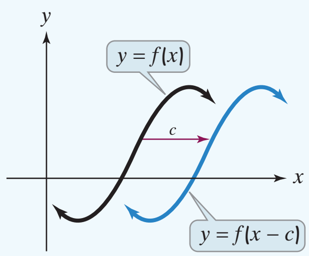
In general, if c is positive, $y = f(x + c)$ shifts the graph of f to the left c units and $y = f(x - c)$ shifts the graph of f to the right c units. These are called **horizontal shifts** of the graph of f.

Horizontal Shifts

Let f be a function and c a positive real number.

- The graph of $y = f(x + c)$ is the graph of $y = f(x)$ shifted to the left c units.
- The graph of $y = f(x - c)$ is the graph of $y = f(x)$ shifted to the right c units.





Study Tip

On a number line, if x represents a number and c is positive, then $x + c$ lies c units to the right of x and $x - c$ lies c units to the left of x. This orientation does not apply to horizontal shifts: $f(x + c)$ causes a shift of c units to the left and $f(x - c)$ causes a shift of c units to the right.

EXAMPLE 2 Horizontal Shift to the Left

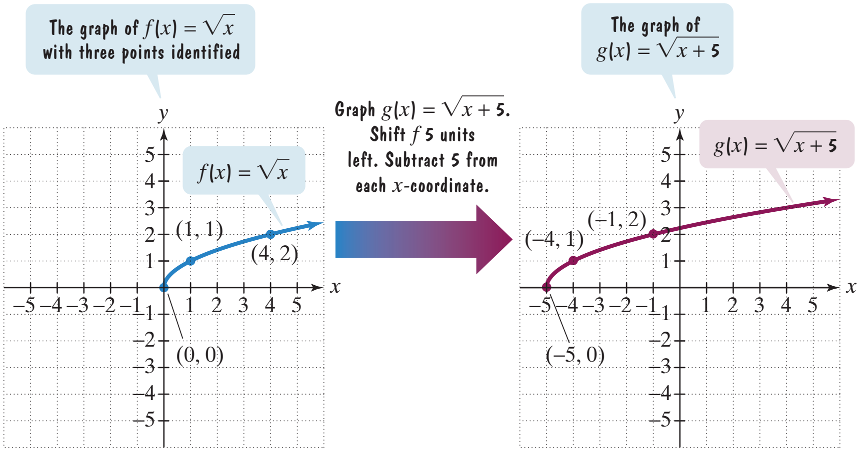
Use the graph of $f(x) = \sqrt{x}$ to obtain the graph of $g(x) = \sqrt{x + 5}$.

Solution Compare the equations for $f(x) = \sqrt{x}$ and $g(x) = \sqrt{x + 5}$. The equation for g adds 5 to each value of x before taking the square root.

$$y = g(x) = \sqrt{x + 5} = f(x + 5)$$

The graph of g shifts the graph of f 5 units to the left.

The graph of $g(x) = \sqrt{x + 5}$ has the same shape as the graph of $f(x) = \sqrt{x}$. However, it is shifted horizontally to the left 5 units.



Study Tip

Notice the difference between $f(x) + c$ and $f(x + c)$.

- $y = f(x) + c$ shifts the graph of $y = f(x)$ c units vertically upward.
- $y = f(x + c)$ shifts the graph of $y = f(x)$ c units horizontally to the left.

There are analogous differences between $f(x) - c$ and $f(x - c)$.

Check Point 2 Use the graph of $f(x) = \sqrt{x}$ to obtain the graph of $g(x) = \sqrt{x - 4}$.

Some functions can be graphed by combining horizontal and vertical shifts. These functions will be variations of a function whose equation you know how to graph, such as the standard quadratic function, the standard cubic function, the square root function, the cube root function, or the absolute value function.

In our next example, we will use the graph of the standard quadratic function, $f(x) = x^2$, to obtain the graph of $h(x) = (x + 1)^2 - 3$. We will graph three functions:

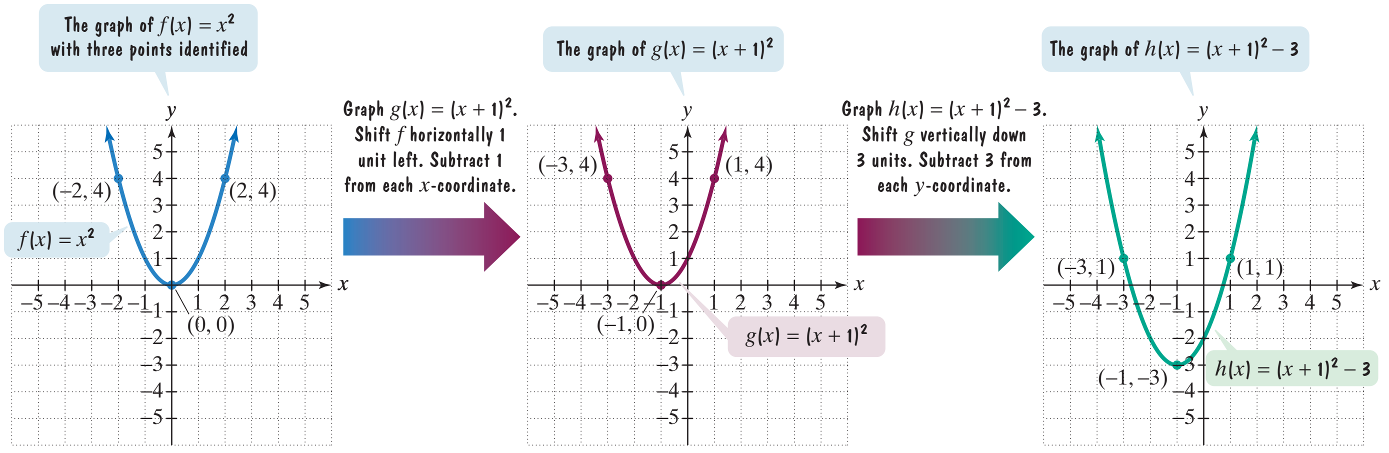
$$f(x) = x^2 \qquad g(x) = (x + 1)^2 \qquad h(x) = (x + 1)^2 - 3.$$

Start by graphing the standard quadratic function. | Shift the graph of f horizontally one unit to the left. | Shift the graph of g vertically down 3 units.

EXAMPLE 3 Combining Horizontal and Vertical Shifts

Use the graph of $f(x) = x^2$ to obtain the graph of $h(x) = (x + 1)^2 - 3$.

Solution



Discovery

Work Example 3 by first shifting the graph of $f(x) = x^2$ three units down, graphing $g(x) = x^2 - 3$. Now, shift this graph one unit left to graph $h(x) = (x + 1)^2 - 3$. Did you obtain the last graph shown in the solution of Example 3? What can you conclude?

Check Point 3 Use the graph of $f(x) = \sqrt{x}$ to obtain the graph of $h(x) = \sqrt{x - 1} - 2$.

4 Use reflections to graph functions.

Reflections of Graphs

This photograph shows a reflection of an old bridge in a Maryland river. This perfect reflection occurs because the surface of the water is absolutely still. A mild breeze rippling the water's surface would distort the reflection.

Is it possible for graphs to have mirror-like qualities? Yes. **Figure 1.57** shows the graphs of $f(x) = x^2$ and $g(x) = -x^2$. The graph of g is a **reflection about the *x*-axis** of the graph of f. For corresponding values of x, the y-coordinates of g are the opposites of the y-coordinates of f. In general, the graph of $y = -f(x)$ reflects the graph of f about the x-axis. Thus, the graph of g is a reflection of the graph of f about the x-axis because

$$g(x) = -x^2 = -f(x).$$

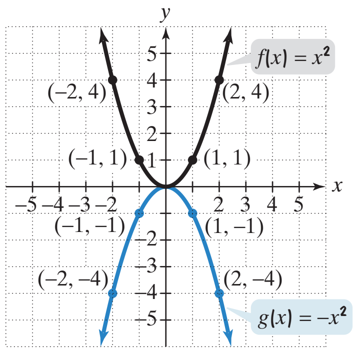


Figure 1.57 Reflection about the x-axis

Reflection about the *x*-Axis

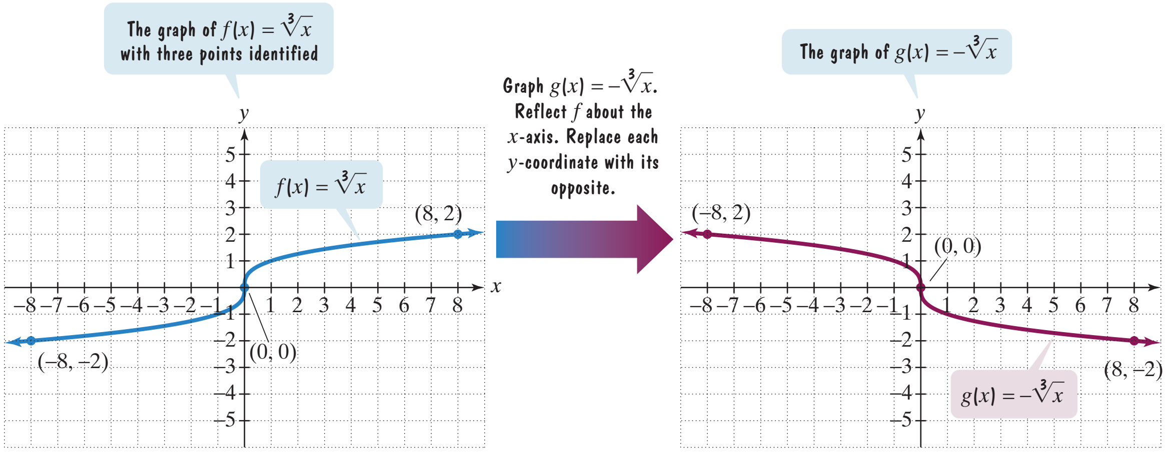
The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected about the x-axis.

EXAMPLE 4 Reflection about the x-Axis

Use the graph of $f(x) = \sqrt[3]{x}$ to obtain the graph of $g(x) = -\sqrt[3]{x}$.

Solution Compare the equations for $f(x) = \sqrt[3]{x}$ and $g(x) = -\sqrt[3]{x}$. The graph of g is a reflection about the x-axis of the graph of f because

$$g(x) = -\sqrt[3]{x} = -f(x).$$



Check Point 4 Use the graph of $f(x) = |x|$ to obtain the graph of $g(x) = -|x|$.

It is also possible to reflect graphs about the y-axis.

Reflection about the y-Axis

The graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected about the y-axis.

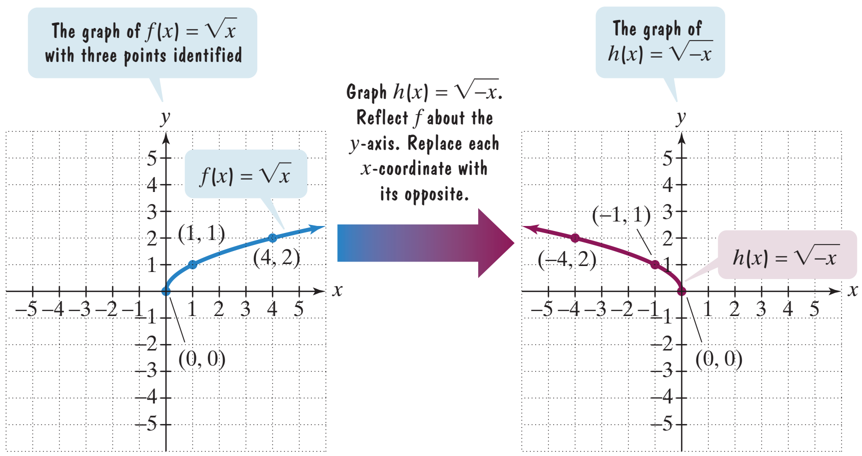
For each point (x, y) on the graph of $y = f(x)$, the point $(-x, y)$ is on the graph of $y = f(-x)$.

EXAMPLE 5 Reflection about the y-Axis

Use the graph of $f(x) = \sqrt{x}$ to obtain the graph of $h(x) = \sqrt{-x}$.

Solution Compare the equations for $f(x) = \sqrt{x}$ and $h(x) = \sqrt{-x}$. The graph of h is a reflection about the y-axis of the graph of f because

$$h(x) = \sqrt{-x} = f(-x).$$



Check Point 5 Use the graph of $f(x) = \sqrt[3]{x}$ to obtain the graph of $h(x) = \sqrt[3]{-x}$.

5 Use vertical stretching and shrinking to graph functions.

Vertical Stretching and Shrinking

Morphing does much more than move an image horizontally, vertically, or about an axis. An object having one shape is transformed into a different shape. Horizontal shifts, vertical shifts, and reflections do not change the basic shape of a graph. Graphs remain rigid and proportionally the same when they undergo these transformations. How can we shrink and stretch graphs, thereby altering their basic shapes?

Look at the three graphs in **Figure 1.58**. The black graph in the middle is the graph of the standard quadratic function, $f(x) = x^2$. Now, look at the blue graph on the top. The equation of this graph is $g(x) = 2x^2$, or $g(x) = 2f(x)$. Thus, for each x, the y-coordinate of g is 2 times as large as the corresponding y-coordinate on the graph of f. The result is a narrower graph because the values of y are rising faster. We say that the graph of g is obtained by vertically *stretching* the graph of f. Now, look at the red graph on the bottom. The equation of this graph is $h(x) = \frac{1}{2}x^2$, or $h(x) = \frac{1}{2}f(x)$. Thus, for each x, the y-coordinate of h is one-half as large as the corresponding y-coordinate on the graph of f. The result is a wider graph because the values of y are rising more slowly. We say that the graph of h is obtained by vertically *shrinking* the graph of f.

These observations can be summarized as follows:

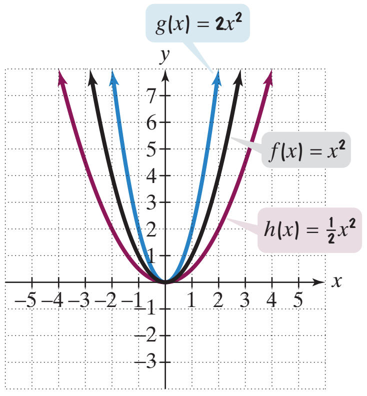
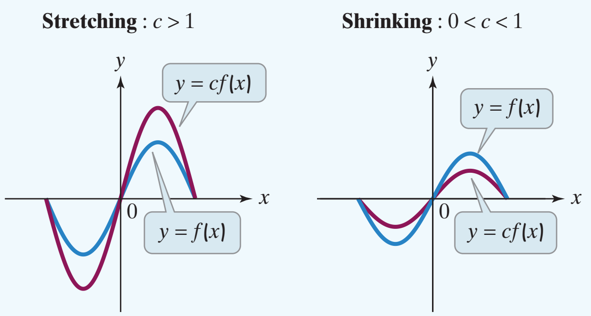


Figure 1.58 Vertically stretching and shrinking $f(x) = x^2$

Vertically Stretching and Shrinking Graphs

Let f be a function and c a positive real number.

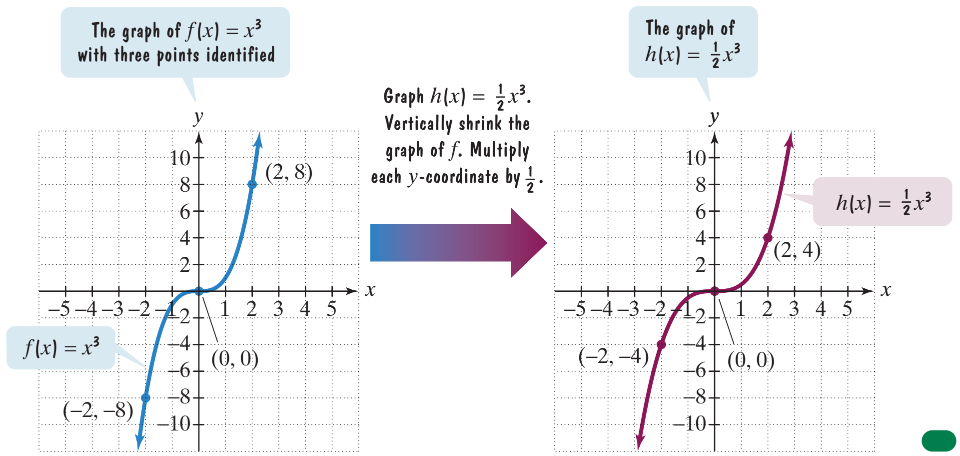
- If $c > 1$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ vertically stretched by multiplying each of its y-coordinates by c.
- If $0 < c < 1$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ vertically shrunk by multiplying each of its y-coordinates by c.



EXAMPLE 6 Vertically Shrinking a Graph

Use the graph of $f(x) = x^3$ to obtain the graph of $h(x) = \frac{1}{2}x^3$.

Solution The graph of $h(x) = \frac{1}{2}x^3$ is obtained by vertically shrinking the graph of $f(x) = x^3$.



Check Point 6 Use the graph of $f(x) = |x|$ to obtain the graph of $g(x) = 2|x|$.

6 Use horizontal stretching and shrinking to graph functions.

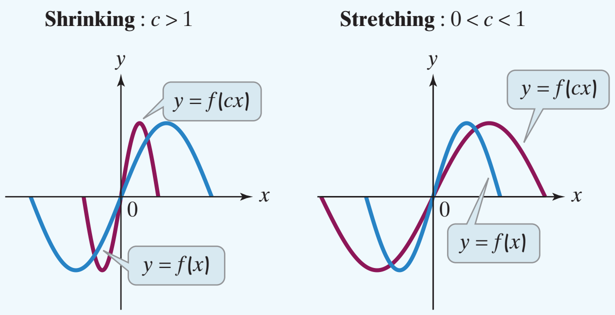
Horizontal Stretching and Shrinking

It is also possible to stretch and shrink graphs horizontally.

Horizontally Stretching and Shrinking Graphs

Let f be a function and c a positive real number.

- If $c > 1$, the graph of $y = f(cx)$ is the graph of $y = f(x)$ horizontally shrunk by dividing each of its x-coordinates by c.
- If $0 < c < 1$, the graph of $y = f(cx)$ is the graph of $y = f(x)$ horizontally stretched by dividing each of its x-coordinates by c.



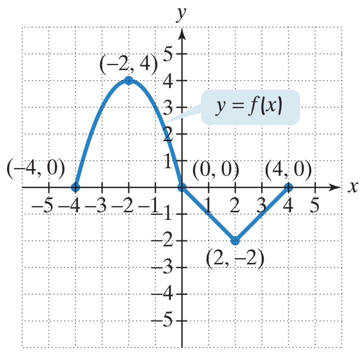


Figure 1.59

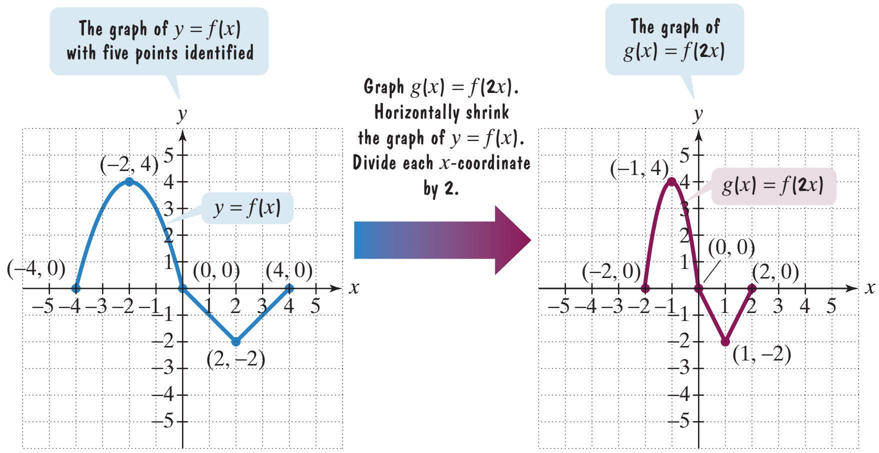
EXAMPLE 7 Horizontally Stretching and Shrinking a Graph

Use the graph of $y = f(x)$ in **Figure 1.59** to obtain each of the following graphs:

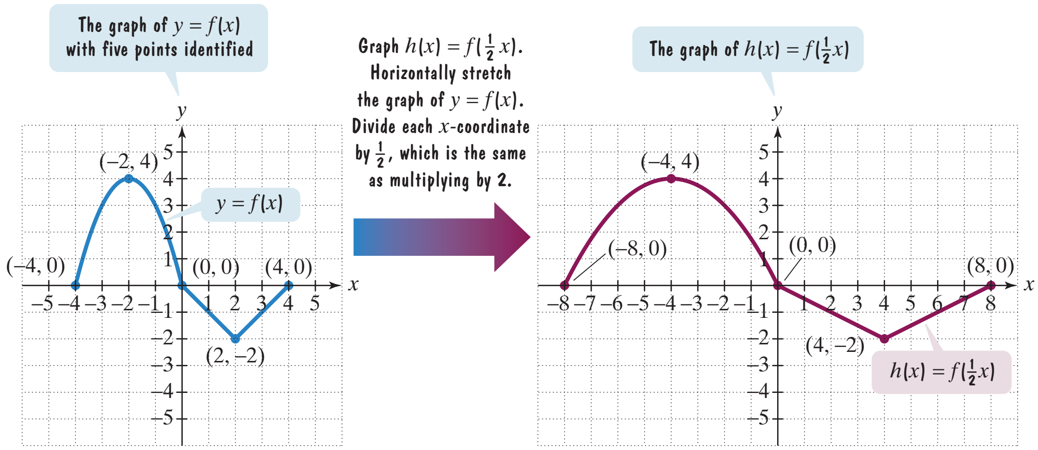
a. $g(x) = f(2x)$ **b.** $h(x) = f\left(\frac{1}{2}x\right)$.

Solution

a. The graph of $g(x) = f(2x)$ is obtained by horizontally shrinking the graph of $y = f(x)$.



b. The graph of $h(x) = f\left(\frac{1}{2}x\right)$ is obtained by horizontally stretching the graph of $y = f(x)$.



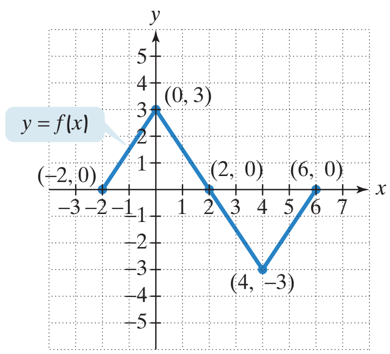


Figure 1.60

Check Point 7 Use the graph of $y = f(x)$ in **Figure 1.60** to obtain each of the following graphs:

a. $g(x) = f(2x)$ **b.** $h(x) = f\left(\frac{1}{2}x\right)$.

7 Graph functions involving a sequence of transformations.

Sequences of Transformations

Table 1.4 summarizes the procedures for transforming the graph of $y = f(x)$.

Table 1.4 Summary of Transformations

In each case, c represents a positive real number.

To Graph:	Draw the Graph of f and:	Changes in the Equation of $y = f(x)$
Vertical shifts $y = f(x) + c$	Raise the graph of f by c units.	c is added to $f(x)$.
$y = f(x) - c$	Lower the graph of f by c units.	c is subtracted from $f(x)$.
Horizontal shifts $y = f(x + c)$	Shift the graph of f to the left c units.	x is replaced with $x + c$.
$y = f(x - c)$	Shift the graph of f to the right c units.	x is replaced with $x - c$.
Reflection about the x-axis $y = -f(x)$	Reflect the graph of f about the x-axis.	$f(x)$ is multiplied by -1.
Reflection about the y-axis $y = f(-x)$	Reflect the graph of f about the y-axis.	x is replaced with $-x$.
Vertical stretching or shrinking $y = cf(x), c > 1$	Multiply each y-coordinate of $y = f(x)$ by c, vertically stretching the graph of f.	$f(x)$ is multiplied by $c, c > 1$.
$y = cf(x), 0 < c < 1$	Multiply each y-coordinate of $y = f(x)$ by c, vertically shrinking the graph of f.	$f(x)$ is multiplied by $c, 0 < c < 1$.
Horizontal stretching or shrinking $y = f(cx), c > 1$	Divide each x-coordinate of $y = f(x)$ by c, horizontally shrinking the graph of f.	x is replaced with $cx, c > 1$.
$y = f(cx), 0 < c < 1$	Divide each x-coordinate of $y = f(x)$ by c, horizontally stretching the graph of f.	x is replaced with $cx, 0 < c < 1$.

A function involving more than one transformation can be graphed by performing transformations in the following order:

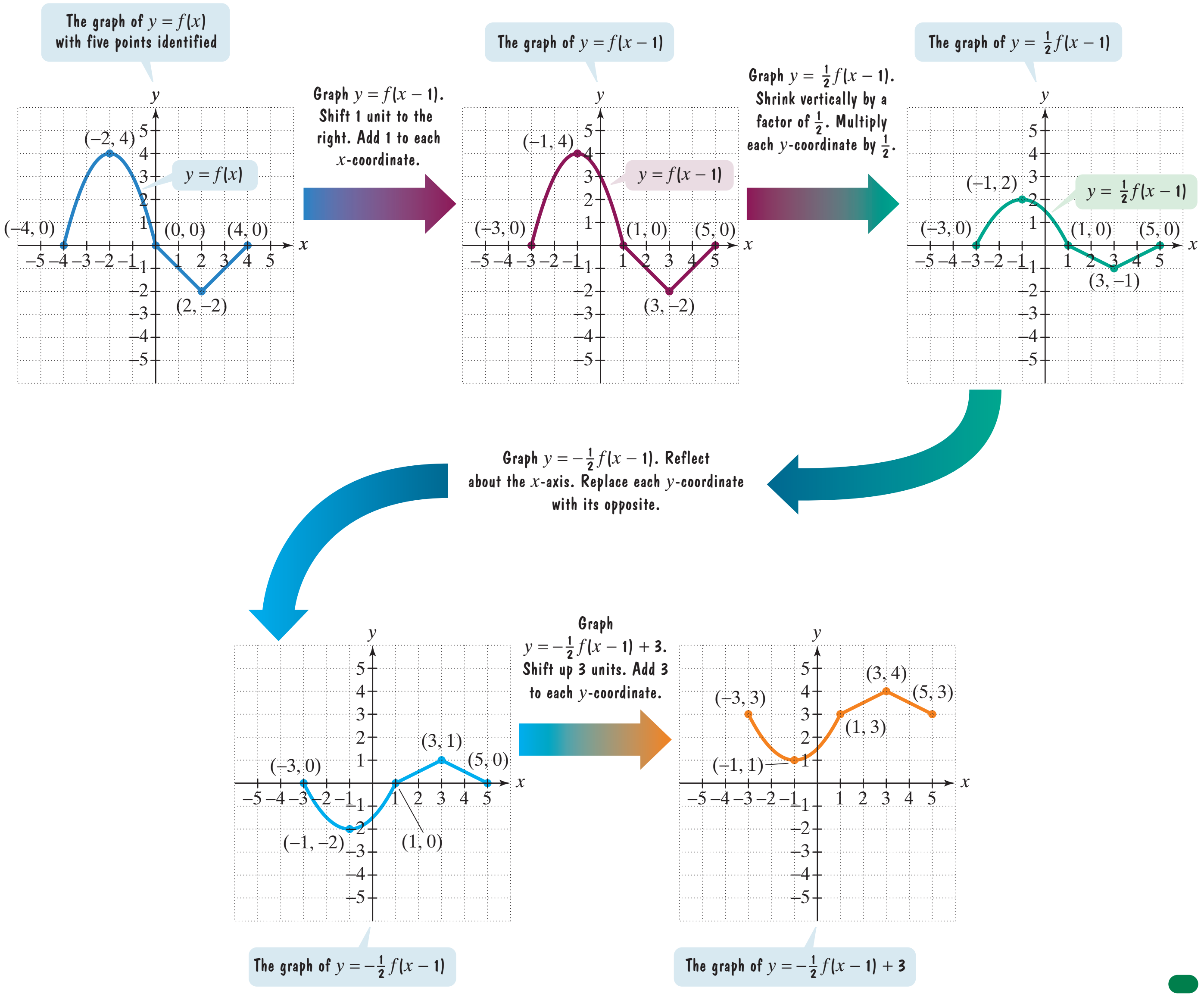
1. Horizontal shifting
2. Stretching or shrinking
3. Reflecting
4. Vertical shifting

EXAMPLE 8 Graphing Using a Sequence of Transformations

Use the graph of $y = f(x)$ given in **Figure 1.59** of Example 7 on page 212, and repeated below, to graph $y = -\frac{1}{2}f(x-1) + 3$.

Solution Our graphs will evolve in the following order:

1. Horizontal shifting: Graph $y = f(x-1)$ by shifting the graph of $y = f(x)$ 1 unit to the right.
2. Shrinking: Graph $y = \frac{1}{2}f(x-1)$ by shrinking the previous graph by a factor of $\frac{1}{2}$.
3. Reflecting: Graph $y = -\frac{1}{2}f(x-1)$ by reflecting the previous graph about the x-axis.
4. Vertical shifting: Graph $y = -\frac{1}{2}f(x-1) + 3$ by shifting the previous graph up 3 units.



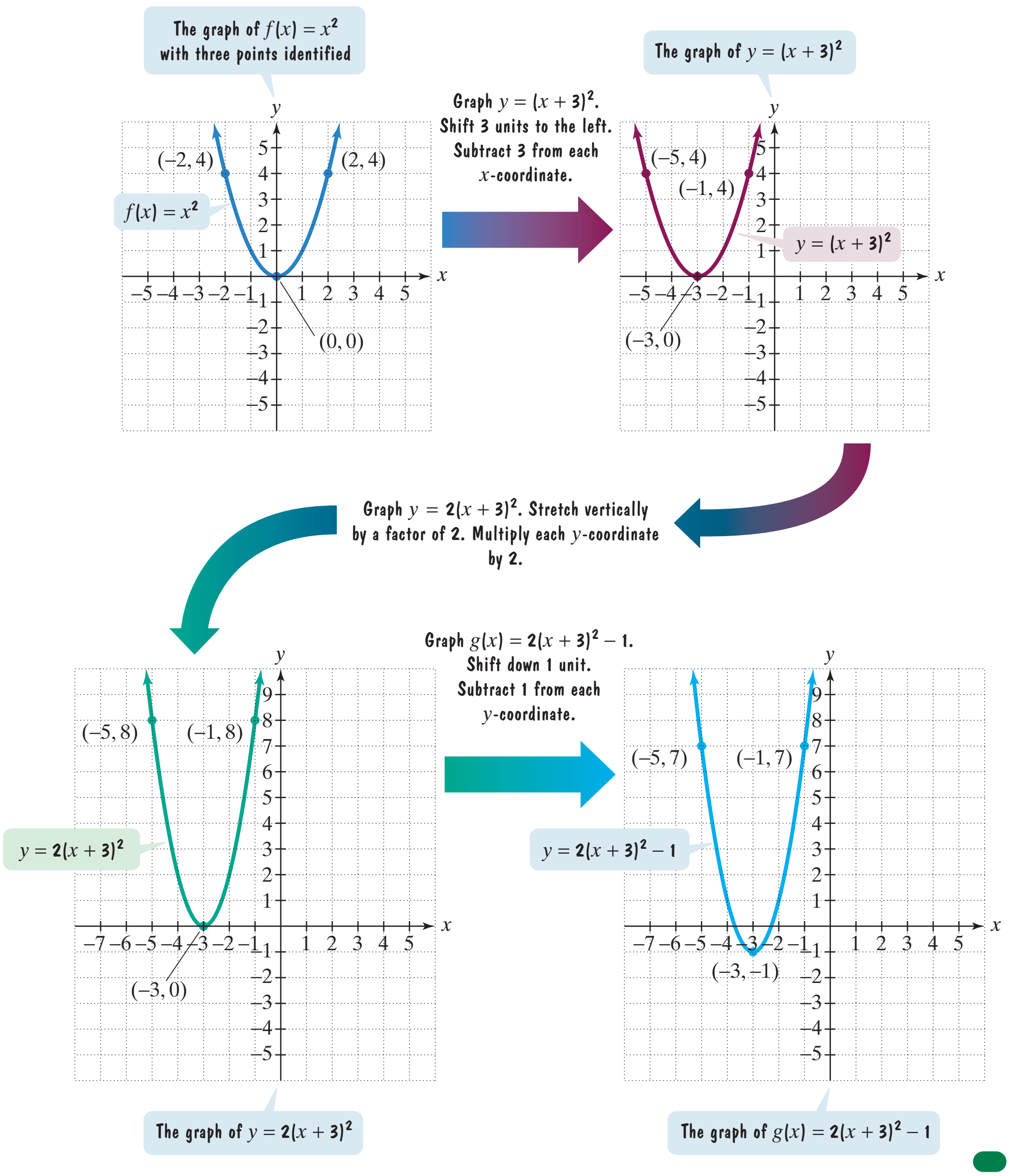
Check Point 8 Use the graph of $y = f(x)$ given in **Figure 1.60** of Check Point 7 on page 213 to graph $y = -\frac{1}{3}f(x + 1) - 2$.

EXAMPLE 9 Graphing Using a Sequence of Transformations

Use the graph of $f(x) = x^2$ to graph $g(x) = 2(x + 3)^2 - 1$.

Solution Our graphs will evolve in the following order:

1. Horizontal shifting: Graph $y = (x + 3)^2$ by shifting the graph of $f(x) = x^2$ three units to the left.
2. Stretching: Graph $y = 2(x + 3)^2$ by stretching the previous graph by a factor of 2.
3. Vertical shifting: Graph $g(x) = 2(x + 3)^2 - 1$ by shifting the previous graph down 1 unit.

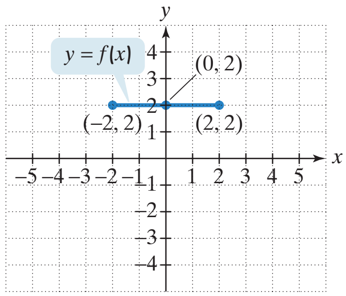


Check Point 9 Use the graph of $f(x) = x^2$ to graph $g(x) = 2(x - 1)^2 + 3$.

Exercise Set 1.6

Practice Exercises

In Exercises 1–16, use the graph of $y = f(x)$ to graph each function g.



1. $g(x) = f(x) + 1$

2. $g(x) = f(x) - 1$

3. $g(x) = f(x + 1)$

4. $g(x) = f(x - 1)$

5. $g(x) = f(x - 1) - 2$

6. $g(x) = f(x + 1) + 2$

7. $g(x) = f(-x)$

8. $g(x) = -f(x)$

9. $g(x) = -f(x) + 3$

10. $g(x) = f(-x) + 3$

11. $g(x) = \frac{1}{2}f(x)$

12. $g(x) = 2f(x)$

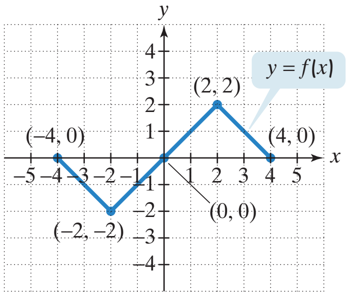
13. $g(x) = f\left(\frac{1}{2}x\right)$

14. $g(x) = f(2x)$

15. $g(x) = -f\left(\frac{1}{2}x\right) + 1$

16. $g(x) = -f(2x) - 1$

In Exercises 17–32, use the graph of $y = f(x)$ to graph each function g.



17. $g(x) = f(x) - 1$

18. $g(x) = f(x) + 1$

19. $g(x) = f(x - 1)$

20. $g(x) = f(x + 1)$

21. $g(x) = f(x - 1) + 2$

22. $g(x) = f(x + 1) - 2$

23. $g(x) = -f(x)$

24. $g(x) = f(-x)$

25. $g(x) = f(-x) + 1$

26. $g(x) = -f(x) + 1$

27. $g(x) = 2f(x)$

28. $g(x) = \frac{1}{2}f(x)$

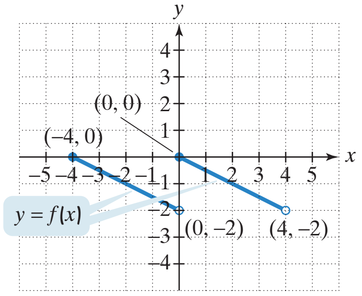
29. $g(x) = f(2x)$

30. $g(x) = f\left(\frac{1}{2}x\right)$

31. $g(x) = 2f(x + 2) + 1$

32. $g(x) = 2f(x + 2) - 1$

In Exercises 33–44, use the graph of $y = f(x)$ to graph each function g.



33. $g(x) = f(x) + 2$

34. $g(x) = f(x) - 2$

35. $g(x) = f(x + 2)$

36. $g(x) = f(x - 2)$

37. $g(x) = -f(x + 2)$

38. $g(x) = -f(x - 2)$

39. $g(x) = -\frac{1}{2}f(x + 2)$

40. $g(x) = -\frac{1}{2}f(x - 2)$

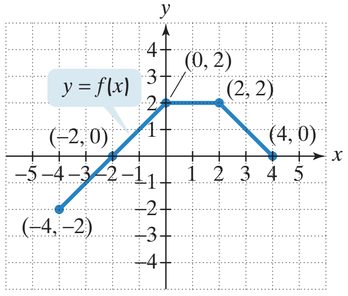
41. $g(x) = -\frac{1}{2}f(x + 2) - 2$

42. $g(x) = -\frac{1}{2}f(x - 2) + 2$

43. $g(x) = \frac{1}{2}f(2x)$

44. $g(x) = 2f\left(\frac{1}{2}x\right)$

In Exercises 45–52, use the graph of $y = f(x)$ to graph each function g.



45. $g(x) = f(x - 1) - 1$

46. $g(x) = f(x + 1) + 1$

47. $g(x) = -f(x - 1) + 1$

48. $g(x) = -f(x + 1) - 1$

49. $g(x) = 2f\left(\frac{1}{2}x\right)$

50. $g(x) = \frac{1}{2}f(2x)$

51. $g(x) = \frac{1}{2}f(x + 1)$

52. $g(x) = 2f(x - 1)$

In Exercises 53–66, begin by graphing the standard quadratic function, $f(x) = x^2$. Then use transformations of this graph to graph the given function.

53. $g(x) = x^2 - 2$

54. $g(x) = x^2 - 1$

55. $g(x) = (x - 2)^2$

56. $g(x) = (x - 1)^2$

57. $h(x) = -(x - 2)^2$

58. $h(x) = -(x - 1)^2$

59. $h(x) = (x - 2)^2 + 1$

60. $h(x) = (x - 1)^2 + 2$

61. $g(x) = 2(x - 2)^2$

62. $g(x) = \frac{1}{2}(x - 1)^2$

63. $h(x) = 2(x - 2)^2 - 1$

64. $h(x) = \frac{1}{2}(x - 1)^2 - 1$

65. $h(x) = -2(x + 1)^2 + 1$

66. $h(x) = -2(x + 2)^2 + 1$

In Exercises 67–80, begin by graphing the square root function, $f(x) = \sqrt{x}$. Then use transformations of this graph to graph the given function.

67. $g(x) = \sqrt{x} + 2$ **68.** $g(x) = \sqrt{x} + 1$

69. $g(x) = \sqrt{x + 2}$ **70.** $g(x) = \sqrt{x + 1}$

71. $h(x) = -\sqrt{x + 2}$ **72.** $h(x) = -\sqrt{x + 1}$

73. $h(x) = \sqrt{-x + 2}$ **74.** $h(x) = \sqrt{-x + 1}$

75. $g(x) = \frac{1}{2}\sqrt{x + 2}$ **76.** $g(x) = 2\sqrt{x + 1}$

77. $h(x) = \sqrt{x + 2} - 2$ **78.** $h(x) = \sqrt{x + 1} - 1$

79. $g(x) = 2\sqrt{x + 2} - 2$ **80.** $g(x) = 2\sqrt{x + 1} - 1$

In Exercises 81–94, begin by graphing the absolute value function, $f(x) = |x|$. Then use transformations of this graph to graph the given function.

81. $g(x) = |x| + 4$ **82.** $g(x) = |x| + 3$

83. $g(x) = |x + 4|$ **84.** $g(x) = |x + 3|$

85. $h(x) = |x + 4| - 2$ **86.** $h(x) = |x + 3| - 2$

87. $h(x) = -|x + 4|$ **88.** $h(x) = -|x + 3|$

89. $g(x) = -|x + 4| + 1$ **90.** $g(x) = -|x + 4| + 2$

91. $h(x) = 2|x + 4|$ **92.** $h(x) = 2|x + 3|$

93. $g(x) = -2|x + 4| + 1$ **94.** $g(x) = -2|x + 3| + 2$

In Exercises 95–106, begin by graphing the standard cubic function, $f(x) = x^3$. Then use transformations of this graph to graph the given function.

95. $g(x) = x^3 - 3$ **96.** $g(x) = x^3 - 2$

97. $g(x) = (x - 3)^3$ **98.** $g(x) = (x - 2)^3$

99. $h(x) = -x^3$ **100.** $h(x) = -(x - 2)^3$

101. $h(x) = \frac{1}{2}x^3$ **102.** $h(x) = \frac{1}{4}x^3$

103. $r(x) = (x - 3)^3 + 2$ **104.** $r(x) = (x - 2)^3 + 1$

105. $h(x) = \frac{1}{2}(x - 3)^3 - 2$ **106.** $h(x) = \frac{1}{2}(x - 2)^3 - 1$

In Exercises 107–118, begin by graphing the cube root function, $f(x) = \sqrt[3]{x}$. Then use transformations of this graph to graph the given function.

107. $g(x) = \sqrt[3]{x} + 2$ **108.** $g(x) = \sqrt[3]{x} - 2$

109. $g(x) = \sqrt[3]{x + 2}$ **110.** $g(x) = \sqrt[3]{x - 2}$

111. $h(x) = \frac{1}{2}\sqrt[3]{x + 2}$ **112.** $h(x) = \frac{1}{2}\sqrt[3]{x - 2}$

113. $r(x) = \frac{1}{2}\sqrt[3]{x + 2} - 2$ **114.** $r(x) = \frac{1}{2}\sqrt[3]{x - 2} + 2$

115. $h(x) = -\sqrt[3]{x + 2}$ **116.** $h(x) = -\sqrt[3]{x - 2}$

117. $g(x) = \sqrt[3]{-x - 2}$ **118.** $g(x) = \sqrt[3]{-x + 2}$

Practice Plus

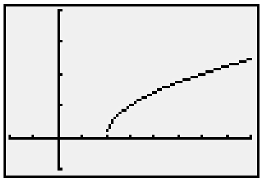
*In Exercises 119–122, use transformations of the graph of the greatest integer function, $f(x) = \text{int}(x)$, to graph each function. (The graph of $f(x) = \text{int}(x)$ is shown in **Figure 1.34** on page 171.)*

119. $g(x) = 2\ \text{int}\ (x + 1)$ **120.** $g(x) = 3\ \text{int}\ (x - 1)$

121. $h(x) = \text{int}\ (-x) + 1$ **122.** $h(x) = \text{int}\ (-x) - 1$

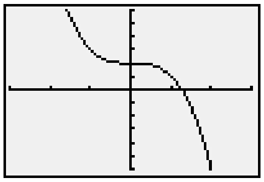
In Exercises 123–126, write a possible equation for the function whose graph is shown. Each graph shows a transformation of a common function.

123.



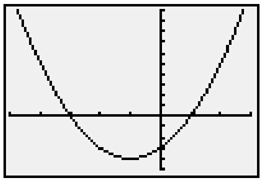
[–2, 8, 1] by [–1, 4, 1]

124.



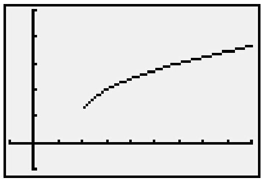
[–3, 3, 1] by [–6, 6, 1]

125.



[–5, 3, 1] by [–5, 10, 1]

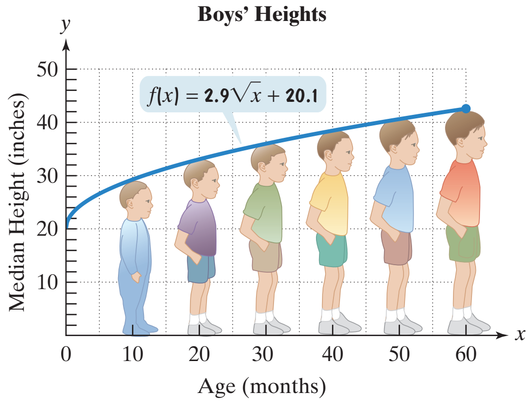
126.



[–1, 9, 1] by [–1, 5, 1]

Application Exercises

127. The function $f(x) = 2.9\sqrt{x} + 20.1$ models the median height, $f(x)$, in inches, of boys who are x months of age. The graph of f is shown.



Source: Laura Walther Nathanson, *The Portable Pediatrician for Parents*

a. Describe how the graph can be obtained using transformations of the square root function $f(x) = \sqrt{x}$.

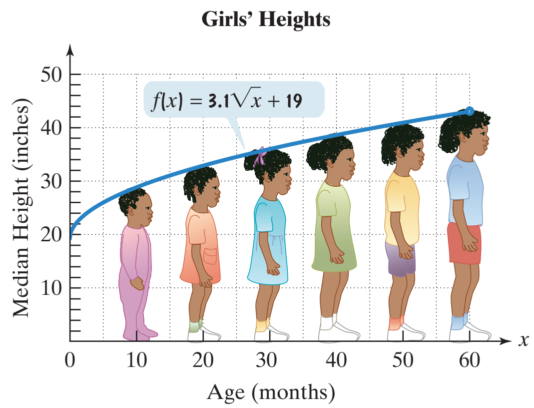
b. According to the model, what is the median height of boys who are 48 months, or four years, old? Use a calculator and round to the nearest tenth of an inch. The actual median height for boys at 48 months is 40.8 inches. How well does the model describe the actual height?

(This exercise continues on the next page.)

c. Use the model to find the average rate of change, in inches per month, between birth and 10 months. Round to the nearest tenth.

d. Use the model to find the average rate of change, in inches per month, between 50 and 60 months. Round to the nearest tenth. How does this compare with your answer in part (c)? How is this difference shown by the graph?

128. The function $f(x) = 3.1\sqrt{x} + 19$ models the median height, $f(x)$, in inches, of girls who are x months of age. The graph of f is shown.



Source: Laura Walther Nathanson, *The Portable Pediatrician for Parents*

a. Describe how the graph can be obtained using transformations of the square root function $f(x) = \sqrt{x}$.

b. According to the model, what is the median height of girls who are 48 months, or four years, old? Use a calculator and round to the nearest tenth of an inch. The actual median height for girls at 48 months is 40.2 inches. How well does the model describe the actual height?

c. Use the model to find the average rate of change, in inches per month, between birth and 10 months. Round to the nearest tenth.

d. Use the model to find the average rate of change, in inches per month, between 50 and 60 months. Round to the nearest tenth. How does this compare with your answer in part (c)? How is this difference shown by the graph?

Writing in Mathematics

129. What must be done to a function's equation so that its graph is shifted vertically upward?

130. What must be done to a function's equation so that its graph is shifted horizontally to the right?

131. What must be done to a function's equation so that its graph is reflected about the x-axis?

132. What must be done to a function's equation so that its graph is reflected about the y-axis?

133. What must be done to a function's equation so that its graph is stretched vertically?

134. What must be done to a function's equation so that its graph is shrunk horizontally?

Technology Exercises

135. a. Use a graphing utility to graph $f(x) = x^2 + 1$.

b. Graph $f(x) = x^2 + 1$, $g(x) = f(2x)$, $h(x) = f(3x)$, and $k(x) = f(4x)$ in the same viewing rectangle.

c. Describe the relationship among the graphs of f, g, h, and k, with emphasis on different values of x for points on all four graphs that give the same y-coordinate.

d. Generalize by describing the relationship between the graph of f and the graph of g, where $g(x) = f(cx)$ for $c > 1$.

e. Try out your generalization by sketching the graphs of $f(cx)$ for $c = 1$, $c = 2$, $c = 3$, and $c = 4$ for a function of your choice.

136. a. Use a graphing utility to graph $f(x) = x^2 + 1$.

b. Graph $f(x) = x^2 + 1$, $g(x) = f\left(\frac{1}{2}x\right)$, and $h(x) = f\left(\frac{1}{4}x\right)$ in the same viewing rectangle.

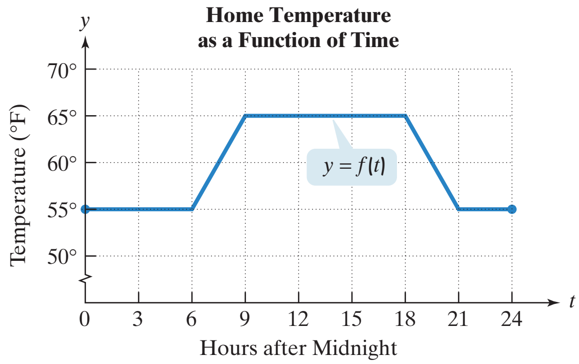
c. Describe the relationship among the graphs of f, g, and h, with emphasis on different values of x for points on all three graphs that give the same y-coordinate.

d. Generalize by describing the relationship between the graph of f and the graph of g, where $g(x) = f(cx)$ for $0 < c < 1$.

e. Try out your generalization by sketching the graphs of $f(cx)$ for $c = 1$, and $c = \frac{1}{2}$, and $c = \frac{1}{4}$ for a function of your choice.

Critical Thinking Exercises

Make Sense? *During the winter, you program your home thermostat so that at midnight, the temperature is 55°. This temperature is maintained until 6 A.M. Then the house begins to warm up so that by 9 A.M. the temperature is 65°. At 6 P.M. the house begins to cool. By 9 P.M., the temperature is again 55°. The graph illustrates home temperature, $f(t)$, as a function of hours after midnight, t.*



In Exercises 137–140, determine whether each statement makes sense or does not make sense, and explain your reasoning. If the statement makes sense, graph the new function on the domain [0, 24]. If the statement does not make sense, correct the function in the statement and graph the corrected function on the domain [0, 24].

137. I decided to keep the house 5° warmer than before, so I reprogrammed the thermostat to $y = f(t) + 5$.

138. I decided to keep the house 5° cooler than before, so I reprogrammed the thermostat to $y = f(t) - 5$.

139. I decided to change the heating schedule to start one hour earlier than before, so I reprogrammed the thermostat to $y = f(t - 1)$.

140. I decided to change the heating schedule to start one hour later than before, so I reprogrammed the thermostat to $y = f(t + 1)$.

In Exercises 141–144, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

141. If $f(x) = |x|$ and $g(x) = |x + 3| + 3$, then the graph of g is a translation of the graph of f three units to the right and three units upward.

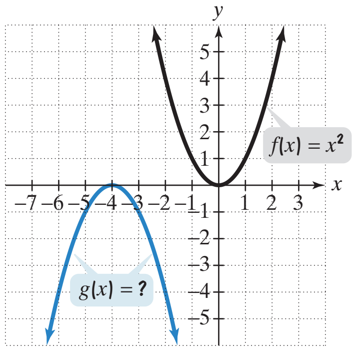
142. If $f(x) = -\sqrt{x}$ and $g(x) = \sqrt{-x}$, then f and g have identical graphs.

143. If $f(x) = x^2$ and $g(x) = 5(x^2 - 2)$, then the graph of g can be obtained from the graph of f by stretching f five units followed by a downward shift of two units.

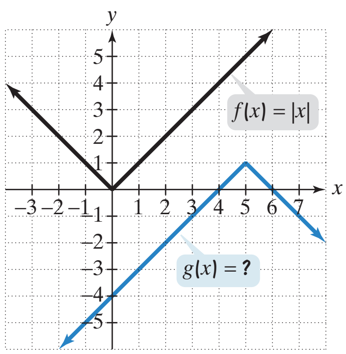
144. If $f(x) = x^3$ and $g(x) = -(x - 3)^3 - 4$, then the graph of g can be obtained from the graph of f by moving f three units to the right, reflecting about the x-axis, and then moving the resulting graph down four units.

In Exercises 145–148, functions f and g are graphed in the same rectangular coordinate system. If g is obtained from f through a sequence of transformations, find an equation for g.

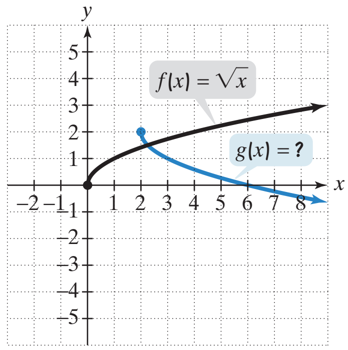
145.



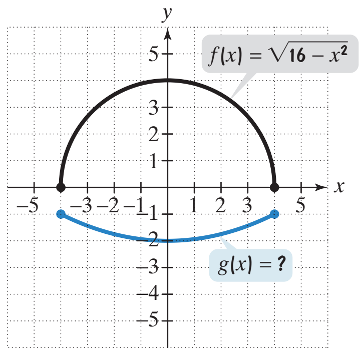
146.



147.



148.



For Exercises 149–152, assume that (a, b) is a point on the graph of f. What is the corresponding point on the graph of each of the following functions?

149. $y = f(-x)$

150. $y = 2f(x)$

151. $y = f(x - 3)$

152. $y = f(x) - 3$

Preview Exercises

Exercises 153–155 will help you prepare for the material covered in the next section.

In Exercises 153–154, perform the indicated operation or operations.

153. $(2x - 1)(x^2 + x - 2)$

154. $(f(x))^2 - 2f(x) + 6$, where $f(x) = 3x - 4$

155. Simplify: $\dfrac{2}{\dfrac{3}{x} - 1}$.