Section 4.8 Applications of Trigonometric Functions

Objectives

1. Solve a right triangle.
2. Solve problems involving bearings.
3. Model simple harmonic motion.

In the late 1960s, popular musicians were searching for new sounds. Film composers were looking for ways to create unique sounds as well. From these efforts, synthesizers that electronically reproduce musical sounds were born. From providing the backbone of today's most popular music to providing the strange sounds for the most experimental music, synthesizing programs now available on computers are at the forefront of music technology.

If we did not understand the periodic nature of sinusoidal functions, the synthesizing programs used in almost all forms of music would not exist. In this section, we look at applications of trigonometric functions in solving right triangles and in modeling periodic phenomena such as sound.

1 Solve a right triangle.

Solving Right Triangles

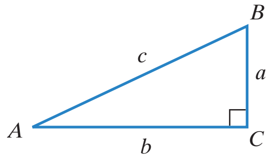


Figure 4.100 Labeling right triangles

Solving a right triangle means finding the missing lengths of its sides and the measurements of its angles. We will label right triangles so that side a is opposite angle A, side b is opposite angle B, and side c, the hypotenuse, is opposite right angle C. **Figure 4.100** illustrates this labeling.

When solving a right triangle, we will use the sine, cosine, and tangent functions, rather than their reciprocals. Example 1 shows how to solve a right triangle when we know the length of a side and the measure of an acute angle.

EXAMPLE 1 Solving a Right Triangle

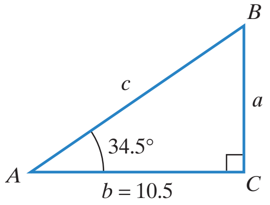


Figure 4.101 Find B, a, and c.

Solve the right triangle shown in **Figure 4.101**, rounding lengths to two decimal places.

Solution We begin by finding the measure of angle B. We do not need a trigonometric function to do so. Because $C = 90°$ and the sum of a triangle's angles is 180°, we see that $A + B = 90°$. Thus,

$$B = 90° - A = 90° - 34.5° = 55.5°.$$

Now we need to find a. Because we have a known angle, an unknown opposite side, and a known adjacent side, we use the tangent function.

$$\tan 34.5° = \frac{a}{10.5}$$

Side opposite the 34.5° angle

Side adjacent to the 34.5° angle

Now we multiply both sides of this equation by 10.5 and solve for a.

$$a = 10.5 \tan 34.5° \approx 7.22$$

Finally, we need to find c. Because we have a known angle, a known adjacent side, and an unknown hypotenuse, we use the cosine function.

$$\cos 34.5° = \frac{10.5}{c}$$

Side adjacent to the 34.5° angle

Hypotenuse

Discovery

There is often more than one correct way to solve a right triangle. In Example 1, find a using angle $B = 55.5°$. Find c using the Pythagorean Theorem.

Now we multiply both sides of $\cos 34.5° = \frac{10.5}{c}$ by c and then solve for c.

$$c \cos 34.5° = 10.5 \quad \text{Multiply both sides by } c.$$

$$c = \frac{10.5}{\cos 34.5°} \approx 12.74 \quad \text{Divide both sides by } \cos 34.5° \text{ and solve for } c.$$

In summary, $B = 55.5°$, $a \approx 7.22$, and $c \approx 12.74$.

Check Point 1 In **Figure 4.100** on the previous page, let $A = 62.7°$ and $a = 8.4$. Solve the right triangle, rounding lengths to two decimal places.

Trigonometry was first developed to measure heights and distances that were inconvenient or impossible to measure directly. In solving application problems, begin by making a sketch involving a right triangle that illustrates the problem's conditions. Then put your knowledge of solving right triangles to work and find the required distance or height.

EXAMPLE 2 Finding a Side of a Right Triangle

From a point on level ground 125 feet from the base of a tower, the angle of elevation is 57.2°. Approximate the height of the tower to the nearest foot.

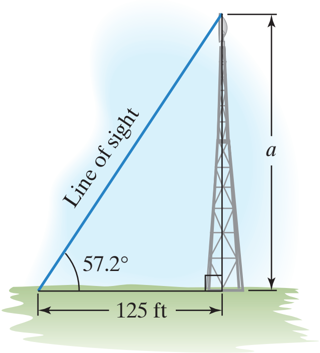


Figure 4.102 Determining height without using direct measurement

Solution A sketch is shown in **Figure 4.102**, where a represents the height of the tower. In the right triangle, we have a known angle, an unknown opposite side, and a known adjacent side. Therefore, we use the tangent function.

$$\tan 57.2° = \frac{a}{125}$$

Side opposite the 57.2° angle

Side adjacent to the 57.2° angle

Now we multiply both sides of this equation by 125 and solve for a.

$$a = 125 \tan 57.2° \approx 194$$

The tower is approximately 194 feet high.

Check Point 2 From a point on level ground 80 feet from the base of the Eiffel Tower, the angle of elevation is 85.4°. Approximate the height of the Eiffel Tower to the nearest foot.

Example 3 illustrates how to find the measure of an acute angle of a right triangle if the lengths of two sides are known.

EXAMPLE 3 Finding an Angle of a Right Triangle

A kite flies at a height of 30 feet when 65 feet of string is out. If the string is in a straight line, find the angle that it makes with the ground. Round to the nearest tenth of a degree.

Solution A sketch is shown in **Figure 4.103**, where A represents the angle the string makes with the ground. In the right triangle, we have an unknown angle, a known opposite side, and a known hypotenuse. Therefore, we use the sine function.

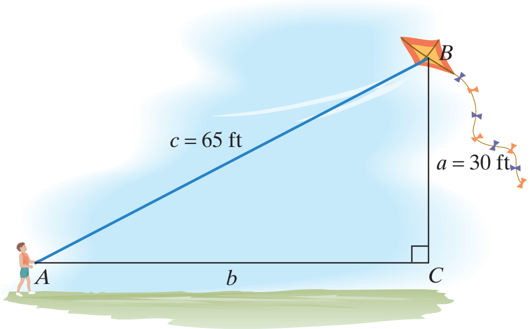


Figure 4.103 Flying a kite

$$\sin A = \frac{30}{65}$$ (Side opposite A / Hypotenuse)

$$A = \sin^{-1}\frac{30}{65} \approx 27.5°$$

The string makes an angle of approximately 27.5° with the ground.

Check Point 3 A guy wire is 13.8 yards long and is attached from the ground to a pole 6.7 yards above the ground. Find the angle, to the nearest tenth of a degree, that the wire makes with the ground.

EXAMPLE 4 Using Two Right Triangles to Solve a Problem

You are taking your first hot-air balloon ride. Your friend is standing on level ground, 100 feet away from your point of launch, making a video of the terrified look on your rapidly ascending face. How rapidly? At one instant, the angle of elevation from the video camera to your face is 31.7°. One minute later, the angle of elevation is 76.2°. How far did you travel, to the nearest tenth of a foot, during that minute?

Solution A sketch that illustrates the problem is shown in **Figure 4.104**. We need to determine $b - a$, the distance traveled during the one-minute period. We find a using the small right triangle. Because we have a known angle, an unknown opposite side, and a known adjacent side, we use the tangent function.

$$\tan 31.7° = \frac{a}{100}$$ (Side opposite the 31.7° angle / Side adjacent to the 31.7° angle)

$$a = 100 \tan 31.7° \approx 61.8$$

We find b using the tangent function in the large right triangle.

$$\tan 76.2° = \frac{b}{100}$$ (Side opposite the 76.2° angle / Side adjacent to the 76.2° angle)

$$b = 100 \tan 76.2° \approx 407.1$$

The balloon traveled 407.1 − 61.8, or approximately 345.3 feet, during the minute.

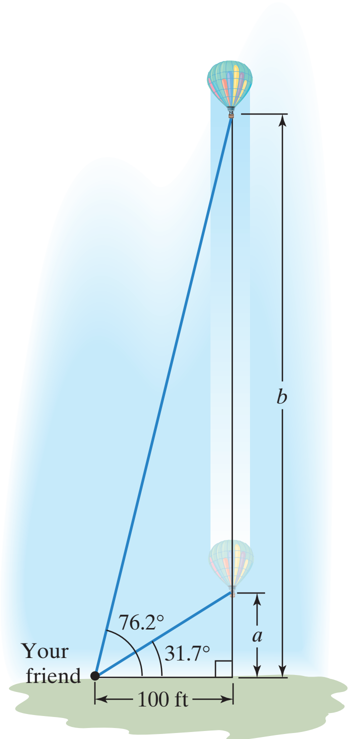


Figure 4.104 Ascending in a hot-air balloon

Check Point 4 You are standing on level ground 800 feet from Mt. Rushmore, looking at the sculpture of Abraham Lincoln's face. The angle of elevation to the bottom of the sculpture is 32° and the angle of elevation to the top is 35°. Find the height of the sculpture of Lincoln's face to the nearest tenth of a foot.

2 Solve problems involving bearings.

Trigonometry and Bearings

In navigation and surveying problems, the term *bearing* is used to specify the location of one point relative to another. The **bearing** from point O to point P is the acute angle, measured in degrees, between ray OP and a north-south line. **Figure 4.105** illustrates some examples of bearings. The north-south line and the east-west line intersect at right angles.

Study Tip

The bearing from O to P can also be described using the phrase "the bearing of P from O."

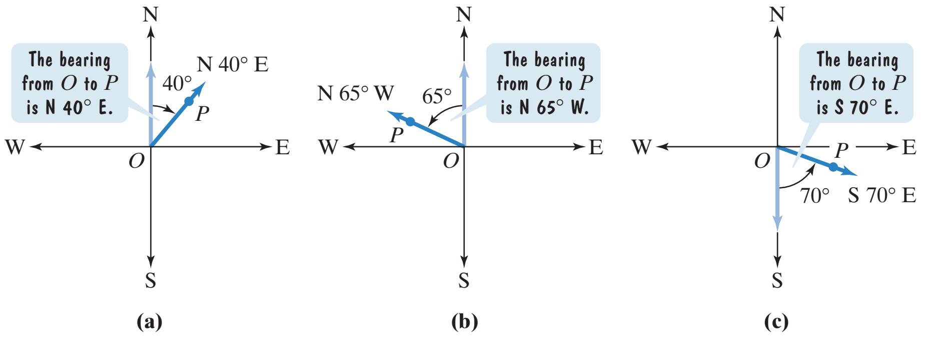


Figure 4.105 An illustration of three bearings

Each bearing has three parts: a letter (N or S), the measure of an acute angle, and a letter (E or W). Here's how we write a bearing:

- If the acute angle is measured from the *north side* of the north-south line, then we write N first. [See **Figure 4.105(a)**.] If the acute angle is measured from the *south side* of the north-south line, then we write S first. [See **Figure 4.105(c)**.]
- Second, we write the measure of the acute angle.
- If the acute angle is measured on the *east side* of the north-south line, then we write E last. [See **Figure 4.105(a)**]. If the acute angle is measured on the *west side* of the north-south line, then we write W last. [See **Figure 4.105(b)**.]

EXAMPLE 5 Understanding Bearings

Use **Figure 4.106** to find each of the following:

a. the bearing from O to B

b. the bearing from O to A.

Solution

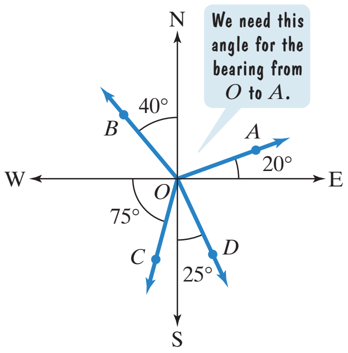


Figure 4.106 Finding bearings

a. To find the bearing from O to B, we need the acute angle between the ray OB and the north-south line through O. The measurement of this angle is given to be 40°. **Figure 4.106** shows that the angle is measured from the north side of the north-south line and lies west of the north-south line. Thus, the bearing from O to B is N 40° W.

b. To find the bearing from O to A, we need the acute angle between the ray OA and the north-south line through O. This angle is specified by the voice balloon in **Figure 4.106**. Because of the given 20° angle, this angle measures $90° - 20°$, or 70°. This angle is measured from the north side of the north-south line. This angle is also east of the north-south line. Thus, the bearing from O to A is N 70° E.

Check Point 5 Use **Figure 4.106** to find each of the following:

a. the bearing from O to D

b. the bearing from O to C.

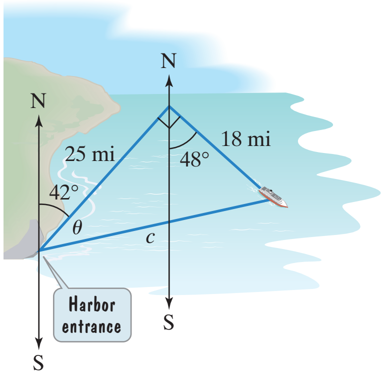


Figure 4.107 Finding a boat's bearing from the harbor entrance

EXAMPLE 6 Finding the Bearing of a Boat

A boat leaves the entrance to a harbor and travels 25 miles on a bearing of N 42° E. **Figure 4.107** shows that the captain then turns the boat 90° clockwise and travels 18 miles on a bearing of S 48° E. At that time:

a. How far is the boat, to the nearest tenth of a mile, from the harbor entrance?

b. What is the bearing, to the nearest tenth of a degree, of the boat from the harbor entrance?

Solution

a. The boat's distance from the harbor entrance is represented by c in **Figure 4.107**. Because we know the length of two sides of the right triangle, we find c using the Pythagorean Theorem. We have

$$c^2 = a^2 + b^2 = 25^2 + 18^2 = 949$$
$$c = \sqrt{949} \approx 30.8.$$

The boat is approximately 30.8 miles from the harbor entrance.

b. The bearing of the boat from the harbor entrance means the bearing from the harbor entrance to the boat. Look at the north-south line passing through the harbor entrance on the left in **Figure 4.107**. The acute angle from this line to the ray on which the boat lies is $42° + \theta$. Because we are measuring the angle from the north side of the line and the boat is east of the harbor, its bearing from the harbor entrance is $\text{N}(42° + \theta)\text{E}$. To find θ, we use the right triangle shown in **Figure 4.107** and the tangent function.

$$\tan\theta = \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta} = \frac{18}{25}$$
$$\theta = \tan^{-1}\frac{18}{25}$$

We can use a calculator in degree mode to find the value of θ: $\theta \approx 35.8°$. Thus, $42° + \theta = 42° + 35.8° = 77.8°$. The bearing of the boat from the harbor entrance is N 77.8° E.

Study Tip

When making a diagram showing bearings, draw a north-south line through each point at which a change in course occurs. The north side of the line lies above each point. The south side of the line lies below each point.

Check Point 6 You leave the entrance to a system of hiking trails and hike 2.3 miles on a bearing of S 31° W. Then the trail turns 90° clockwise and you hike 3.5 miles on a bearing of N 59° W. At that time:

a. How far are you, to the nearest tenth of a mile, from the entrance to the trail system?

b. What is your bearing, to the nearest tenth of a degree, from the entrance to the trail system?

3 Model simple harmonic motion.

Simple Harmonic Motion

Because of their periodic nature, trigonometric functions are used to model phenomena that occur again and again. This includes vibratory or oscillatory motion, such as the motion of a vibrating guitar string, the swinging of a pendulum, or the bobbing of an object attached to a spring. Trigonometric functions are also used to describe radio waves from your favorite FM station, television waves from your not-to-be-missed weekly sitcom, and sound waves from your most-prized CDs.

To see how trigonometric functions are used to model vibratory motion, consider this: A ball is attached to a spring hung from the ceiling. You pull the ball down 4 inches and then release it. If we neglect the effects of friction and air resistance, the ball will continue bobbing up and down on the end of the spring. These up-and-down oscillations are called **simple harmonic motion**.

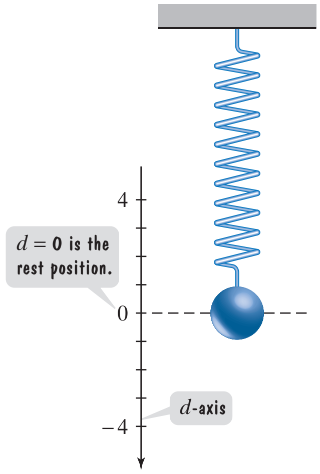


Figure 4.108 Using a d-axis to describe a ball's distance from its rest position

To better understand this motion, we use a d-axis, where d represents distance. This axis is shown in **Figure 4.108**. On this axis, the position of the ball before you pull it down is $d = 0$. This rest position is called the **equilibrium position**. Now you pull the ball down 4 inches to $d = -4$ and release it. **Figure 4.109** shows a sequence of "photographs" taken at one-second time intervals illustrating the distance of the ball from its rest position, d.

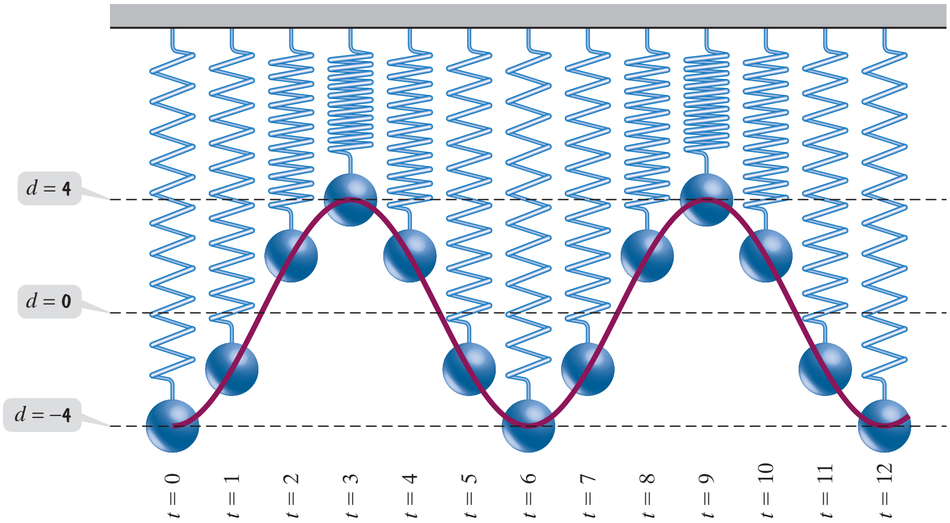


Figure 4.109 A sequence of "photographs" showing the bobbing ball's distance from the rest position, taken at one-second intervals

The curve in **Figure 4.109** shows how the ball's distance from its rest position changes over time. The curve is sinusoidal and the motion can be described using a cosine or a sine function.

Simple Harmonic Motion

An object that moves on a coordinate axis is in **simple harmonic motion** if its distance from the origin, d, at time t is given by either

$$d = a\cos\omega t \quad \text{or} \quad d = a\sin\omega t.$$

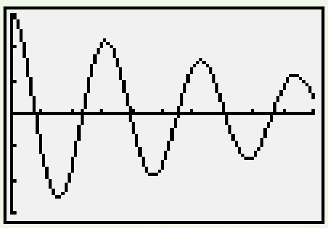
The motion has **amplitude** $|a|$, the maximum displacement of the object from its rest position. The **period** of the motion is $\frac{2\pi}{\omega}$, where $\omega > 0$. The period gives the time it takes for the motion to go through one complete cycle.

Diminishing Motion with Increasing Time

Due to friction and other resistive forces, the motion of an oscillating object decreases over time. The function

$$d = 3e^{-0.1t}\cos 2t$$

models this type of motion. The graph of the function is shown in a $t = [0, 10, 1]$ by $d = [-3, 3, 1]$ viewing rectangle. Notice how the amplitude is decreasing with time as the moving object loses energy.



In describing simple harmonic motion, the equation with the cosine function, $d = a\cos\omega t$, is used if the object is at its greatest distance from rest position, the origin, at $t = 0$. By contrast, the equation with the sine function, $d = a\sin\omega t$, is used if the object is at its rest position, the origin, at $t = 0$.

EXAMPLE 7 Finding an Equation for an Object in Simple Harmonic Motion

A ball on a spring is pulled 4 inches below its rest position and then released. The period of the motion is 6 seconds. Write the equation for the ball's simple harmonic motion.

Solution We need to write an equation that describes d, the distance of the ball from its rest position, after t seconds. (The motion is illustrated by the "photo" sequence in **Figure 4.109**.) When the object is released ($t = 0$), the ball's distance from its rest position is 4 inches down. Because it is *down* 4 inches, d is negative:

When $t = 0$, $d = -4$. Notice that the greatest distance from rest position occurs at $t = 0$. Thus, we will use the equation with the cosine function,

$$d = a \cos \omega t,$$

to model the ball's simple harmonic motion.

Now we determine values for a and ω. Recall that $|a|$ is the maximum displacement. Because the ball is initially below rest position, $a = -4$.

The value of ω in $d = a \cos \omega t$ can be found using the formula for the period.

$$\text{period} = \frac{2\pi}{\omega} = 6 \quad \text{We are given that the period of the motion is 6 seconds.}$$

$$2\pi = 6\omega \quad \text{Multiply both sides by } \omega.$$

$$\omega = \frac{2\pi}{6} = \frac{\pi}{3} \quad \text{Divide both sides by 6 and solve for } \omega.$$

We see that $a = -4$ and $\omega = \frac{\pi}{3}$. Substitute these values into $d = a \cos \omega t$. The equation for the ball's simple harmonic motion is

$$d = -4 \cos \frac{\pi}{3} t.$$

Modeling Music

Sounds are caused by vibrating objects that result in variations in pressure in the surrounding air. Areas of high and low pressure moving through the air are modeled by the harmonic motion formulas. When these vibrations reach our eardrums, the eardrums' vibrations send signals to our brains which create the sensation of hearing.

French mathematician John Fourier (1768–1830) proved that all musical sounds—instrumental and vocal—could be modeled by sums involving sine functions. Modeling musical sounds with sinusoidal functions is used by synthesizing programs available on computers to electronically produce sounds unobtainable from ordinary musical instruments.

Check Point 7 A ball on a spring is pulled 6 inches below its rest position and then released. The period for the motion is 4 seconds. Write the equation for the ball's simple harmonic motion.

The period of the harmonic motion in Example 7 was 6 seconds. It takes 6 seconds for the moving object to complete one cycle. Thus, $\frac{1}{6}$ of a cycle is completed every second. We call $\frac{1}{6}$ the *frequency* of the moving object. **Frequency** describes the number of complete cycles per unit time and is the reciprocal of the period.

Frequency of an Object in Simple Harmonic Motion

An object in simple harmonic motion given by

$$d = a \cos \omega t \quad \text{or} \quad d = a \sin \omega t$$

has **frequency** f given by

$$f = \frac{\omega}{2\pi}, \omega > 0.$$

Equivalently,

$$f = \frac{1}{\text{period}}.$$

EXAMPLE 8 Analyzing Simple Harmonic Motion

Figure 4.110 shows a mass on a smooth surface attached to a spring. The mass moves in simple harmonic motion described by

$$d = 10 \cos \frac{\pi}{6} t,$$

with t measured in seconds and d in centimeters. Find:

a. the maximum displacement

b. the frequency

c. the time required for one cycle.

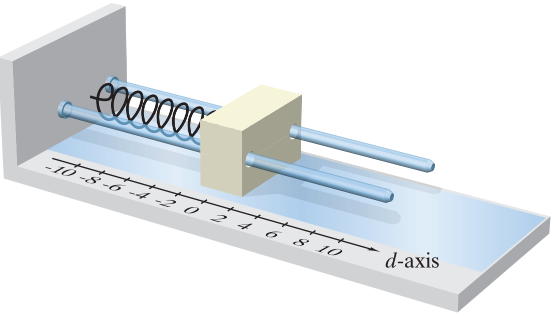


Figure 4.110 A mass attached to a spring, moving in simple harmonic motion

Solution We begin by identifying values for a and ω.

$$d = 10 \cos \frac{\pi}{6} t$$

The form of this equation is $d = a \cos \omega t$ with $a = 10$ and $\omega = \frac{\pi}{6}$.

a. The maximum displacement from the rest position is the amplitude. Because $a = 10$, the maximum displacement is 10 centimeters.

b. The frequency, f, is

$$f = \frac{\omega}{2\pi} = \frac{\frac{\pi}{6}}{2\pi} = \frac{\pi}{6} \cdot \frac{1}{2\pi} = \frac{1}{12}.$$

The frequency is $\frac{1}{12}$ cycle (or oscillation) per second.

c. The time required for one cycle is the period.

$$\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{6}} = 2\pi \cdot \frac{6}{\pi} = 12$$

The time required for one cycle is 12 seconds. This value can also be obtained by taking the reciprocal of the frequency in part (b).

Check Point 8 An object moves in simple harmonic motion described by $d = 12 \cos \frac{\pi}{4} t$, where t is measured in seconds and d in centimeters. Find **a.** the maximum displacement, **b.** the frequency, and **c.** the time required for one cycle.

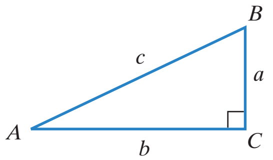
Resisting Damage of Simple Harmonic Motion

Simple harmonic motion from an earthquake caused this highway in Oakland, California, to collapse. By studying the harmonic motion of the soil under the highway, engineers learn to build structures that can resist damage.

Exercise Set 4.8

Practice Exercises

In Exercises 1–12, solve the right triangle shown in the figure. Round lengths to two decimal places and express angles to the nearest tenth of a degree.



1. $A = 23.5°, b = 10$

2. $A = 41.5°, b = 20$

3. $A = 52.6°, c = 54$

4. $A = 54.8°, c = 80$

5. $B = 16.8°, b = 30.5$

6. $B = 23.8°, b = 40.5$

7. $a = 30.4, c = 50.2$

8. $a = 11.2, c = 65.8$

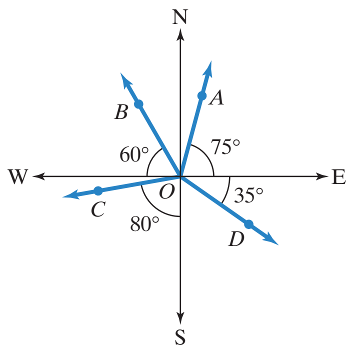
9. $a = 10.8, b = 24.7$

10. $a = 15.3, b = 17.6$

11. $b = 2, c = 7$

12. $b = 4, c = 9$

Use the figure shown to solve Exercises 13–16.



13. Find the bearing from O to A.

14. Find the bearing from O to B.

15. Find the bearing from O to C.

16. Find the bearing from O to D.

In Exercises 17–20, an object is attached to a coiled spring. In Exercises 17–18, the object is pulled down (negative direction from the rest position) and then released. In Exercises 19–20, the object is propelled downward from its rest position at time $t = 0$. Write an equation for the distance of the object from its rest position after t seconds.

Distance from Rest Position at $t = 0$	Amplitude	Period
17. 6 centimeters	6 centimeters	4 seconds
18. 8 inches	8 inches	2 seconds
19. 0 inches	3 inches	1.5 seconds
20. 0 centimeters	5 centimeters	2.5 seconds

In Exercises 21–28, an object moves in simple harmonic motion described by the given equation, where t is measured in seconds and d in inches. In each exercise, find the following:

a. *the maximum displacement*
b. *the frequency*
c. *the time required for one cycle.*

21. $d = 5\cos\frac{\pi}{2}t$

22. $d = 10\cos 2\pi t$

23. $d = -6\cos 2\pi t$

24. $d = -8\cos\frac{\pi}{2}t$

25. $d = \frac{1}{2}\sin 2t$

26. $d = \frac{1}{3}\sin 2t$

27. $d = -5\sin\frac{2\pi}{3}t$

28. $d = -4\sin\frac{3\pi}{2}t$

Practice Plus

In Exercises 29–36, find the length x to the nearest whole number.

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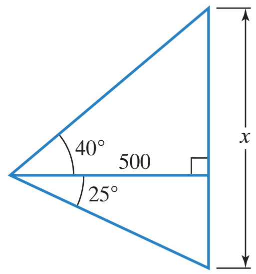

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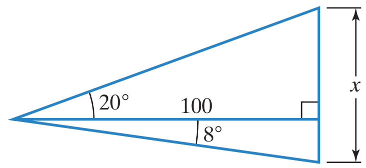

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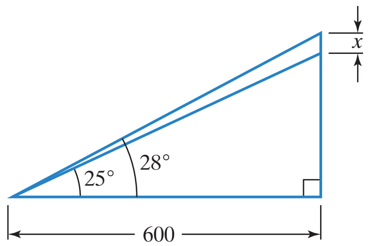

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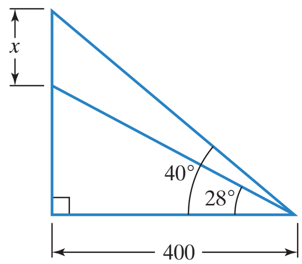

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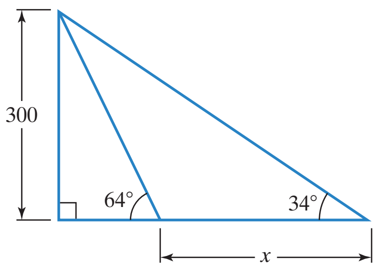

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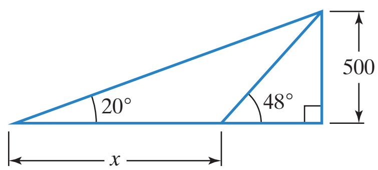

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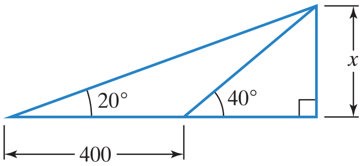
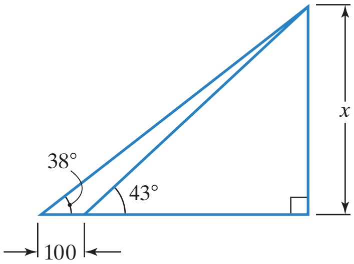


36.



In Exercises 37–40, an object moves in simple harmonic motion described by the given equation, where t is measured in seconds and d in inches. In each exercise, graph one period of the equation. Then find the following:

- **a.** *the maximum displacement*
- **b.** *the frequency*
- **c.** *the time required for one cycle*
- **d.** *the phase shift of the motion.*

Describe how (a) through (d) are illustrated by your graph.

37. $d = 4\cos\left(\pi t - \frac{\pi}{2}\right)$

38. $d = 3\cos\left(\pi t + \frac{\pi}{2}\right)$

39. $d = -2\sin\left(\frac{\pi t}{4} + \frac{\pi}{2}\right)$

40. $d = -\frac{1}{2}\sin\left(\frac{\pi t}{4} - \frac{\pi}{2}\right)$

Application Exercises

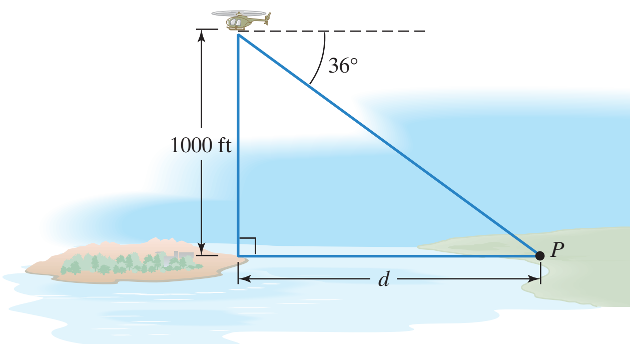
41. The tallest television transmitting tower in the world is in North Dakota. From a point on level ground 5280 feet (one mile) from the base of the tower, the angle of elevation is 21.3°. Approximate the height of the tower to the nearest foot.

42. From a point on level ground 30 yards from the base of a building, the angle of elevation is 38.7°. Approximate the height of the building to the nearest foot.

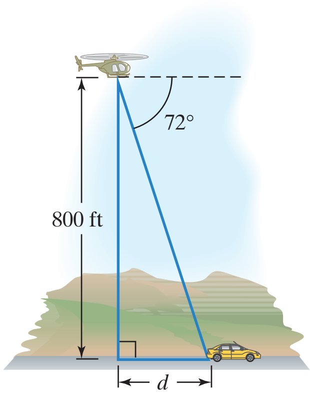
43. The Statue of Liberty is approximately 305 feet tall. If the angle of elevation from a ship to the top of the statue is 23.7°, how far, to the nearest foot, is the ship from the statue's base?

44. A 200-foot cliff drops vertically into the ocean. If the angle of elevation from a ship to the top of the cliff is 22.3°, how far off shore, to the nearest foot, is the ship?

45. A helicopter hovers 1000 feet above a small island. The figure shows that the angle of depression from the helicopter to point P on the coast is 36°. How far off the coast, to the nearest foot, is the island?



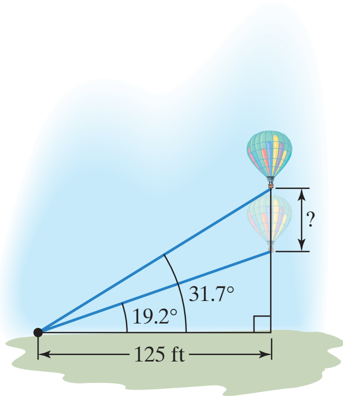
46. A police helicopter is flying at 800 feet. A stolen car is sighted at an angle of depression of 72°. Find the distance of the stolen car, to the nearest foot, from a point directly below the helicopter.



47. A wheelchair ramp is to be built beside the steps to the campus library. Find the angle of elevation of the 23-foot ramp, to the nearest tenth of a degree, if its final height is 6 feet.

48. A building that is 250 feet high casts a shadow 40 feet long. Find the angle of elevation, to the nearest tenth of a degree, of the sun at this time.

49. A hot-air balloon is rising vertically. From a point on level ground 125 feet from the point directly under the passenger compartment, the angle of elevation to the ballon changes from 19.2° to 31.7°. How far, to the nearest tenth of a foot, does the balloon rise during this period?



50. A flagpole is situated on top of a building. The angle of elevation from a point on level ground 330 feet from the building to the top of the flagpole is 63°. The angle of elevation from the same point to the bottom of the flagpole is 53°. Find the height of the flagpole to the nearest tenth of a foot.

51. A boat leaves the entrance to a harbor and travels 150 miles on a bearing of N53°E. How many miles north and how many miles east from the harbor has the boat traveled?

52. A boat leaves the entrance to a harbor and travels 40 miles on a bearing of S64°E. How many miles south and how many miles east from the harbor has the boat traveled?

53. A forest ranger sights a fire directly to the south. A second ranger, 7 miles east of the first ranger, also sights the fire. The bearing from the second ranger to the fire is S 28° W. How far, to the nearest tenth of a mile, is the first ranger from the fire?

54. A ship sights a lighthouse directly to the south. A second ship, 9 miles east of the first ship, also sights the lighthouse. The bearing from the second ship to the lighthouse is S 34° W. How far, to the nearest tenth of a mile, is the first ship from the lighthouse?

55. You leave your house and run 2 miles due west followed by 1.5 miles due north. At that time, what is your bearing from your house?

56. A ship is 9 miles east and 6 miles south of a harbor. What bearing should be taken to sail directly to the harbor?

57. A jet leaves a runway whose bearing is N 35° E from the control tower. After flying 5 miles, the jet turns 90° and files on a bearing of S 55° E for 7 miles. At that time, what is the bearing of the jet from the control tower?

58. A ship leaves port with a bearing of S 40° W. After traveling 7 miles, the ship turns 90° and travels on a bearing of N 50° W for 11 miles. At that time, what is the bearing of the ship from port?

59. An object in simple harmonic motion has a frequency of $\frac{1}{2}$ oscillation per minute and an amplitude of 6 feet. Write an equation in the form $d = a \sin \omega t$ for the object's simple harmonic motion.

60. An object in simple harmonic motion has a frequency of $\frac{1}{4}$ oscillation per minute and an amplitude of 8 feet. Write an equation in the form $d = a \sin \omega t$ for the object's simple harmonic motion.

61. A piano tuner uses a tuning fork. If middle C has a frequency of 264 vibrations per second, write an equation in the form $d = \sin \omega t$ for the simple harmonic motion.

62. A radio station, 98.1 on the FM dial, has radio waves with a frequency of 98.1 million cycles per second. Write an equation in the form $d = \sin \omega t$ for the simple harmonic motion of the radio waves.

Writing in Mathematics

63. What does it mean to solve a right triangle?

64. Explain how to find one of the acute angles of a right triangle if two sides are known.

65. Describe a situation in which a right triangle and a trigonometric function are used to measure a height or distance that would otherwise be inconvenient or impossible to measure.

66. What is meant by the bearing from point O to point P? Give an example with your description.

67. What is simple harmonic motion? Give an example with your description.

68. Explain the period and the frequency of simple harmonic motion. How are they related?

69. Explain how the photograph of the damaged highway on page 573 illustrates simple harmonic motion.

Technology Exercises

The functions in Exercises 70–71 model motion in which the amplitude decreases with time due to friction or other resistive forces. Graph each function in the given viewing rectangle. How many complete oscillations occur on the time interval $0 \le x \le 10$?

70. $y = 4e^{-0.1x} \cos 2x$; $[0, 10, 1]$ by $[-4, 4, 1]$

71. $y = -6e^{-0.09x} \cos 2\pi x$; $[0, 10, 1]$ by $[-6, 6, 1]$

Critical Thinking Exercises

Make Sense? *In Exercises 72–75, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

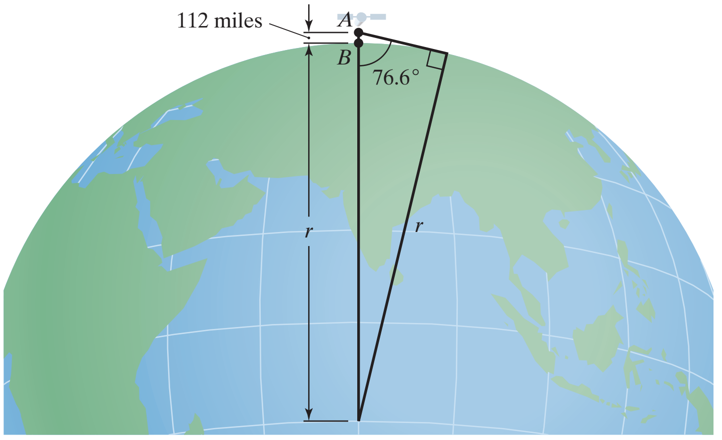
72. A wheelchair ramp must be constructed so that the slope is not more than 1 inch of rise for every 1 foot of run, so I used the tangent function to determine the maximum angle that the ramp can make with the ground.

73. The bearing from O to A is N 103° W.

74. The bearing from O to B is E 70° S.

75. I analyzed simple harmonic motion in which the period was 10 seconds and the frequency was 0.2 oscillation per second.

76. The figure shows a satellite circling 112 miles above Earth. When the satellite is directly above point B, angle A measures 76.6°. Find Earth's radius to the nearest mile.



77. The angle of elevation to the top of a building changes from 20° to 40° as an observer advances 75 feet toward the building. Find the height of the building to the nearest foot.

Group Exercise

78. Music and mathematics have been linked over the centuries. Group members should research and present a seminar to the class on music and mathematics. Be sure to include the role of trigonometric functions in the music-mathematics link.

Preview Exercises

Exercises 79–81 will help you prepare for the material covered in the first section of the next chapter. The exercises use identities, introduced in Section 4.2, that enable you to rewrite trigonometric expressions so that they contain only sines and cosines:

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x}$$
$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}.$$

In Exercises 79–81, rewrite each expression by changing to sines and cosines. Then simplify the resulting expression.

79. $\sec x \cot x$

80. $\tan x \csc x \cos x$

81. $\sec x + \tan x$