109. $\cos^2\left(\frac{1}{2}\sin^{-1}\frac{3}{5}\right)$

110. $\sin^2\left(\frac{1}{2}\cos^{-1}\frac{3}{5}\right)$

111. Use a right triangle to write $\sin(2\sin^{-1}x)$ as an algebraic expression. Assume that x is positive and in the domain of the given inverse trigonometric function.

112. Use the power-reducing formulas to rewrite $\sin^6 x$ as an equivalent expression that does not contain powers of trigonometric functions greater than 1.

Preview Exercises

Exercises 113–115 will help you prepare for the material covered in the next section. In each exercise, use exact values of trigonometric functions to show that the statement is true. Notice that each statement expresses the product of sines and/or cosines as a sum or a difference.

113. $\sin 60° \sin 30° = \frac{1}{2}[\cos(60° - 30°) - \cos(60° + 30°)]$

114. $\cos\frac{\pi}{2}\cos\frac{\pi}{3} = \frac{1}{2}\left[\cos\left(\frac{\pi}{2} - \frac{\pi}{3}\right) + \cos\left(\frac{\pi}{2} + \frac{\pi}{3}\right)\right]$

115. $\sin\pi\cos\frac{\pi}{2} = \frac{1}{2}\left[\sin\left(\pi + \frac{\pi}{2}\right) + \sin\left(\pi - \frac{\pi}{2}\right)\right]$

Chapter 5 Mid-Chapter Check Point

What you Know: Verifying an identity means showing that the expressions on each side are identical. Like solving puzzles, the process can be intriguing because there are sometimes several "best" ways to proceed. We presented some guidelines to help you get started (see page 593). We used fundamental trigonometric identities (see page 586), as well as sum and difference formulas, double-angle formulas, power-reducing formulas, and half-angle formulas (see page 614) to verify identities. We also used these formulas to find exact values of trigonometric functions.

Study Tip

Make copies of the boxes on pages 586 and 614 that contain the essential trigonometric identities. Mount these boxes on cardstock and add this reference sheet to the one you prepared for Chapter 4. (If you didn't prepare a reference sheet for Chapter 4, it's not too late: See the study tip on page 580.)

In Exercises 1–18, verify each identity.

1. $\cos x(\tan x + \cot x) = \csc x$

2. $\dfrac{\sin(x+\pi)}{\cos\left(x + \frac{3\pi}{2}\right)} = \tan^2 x - \sec^2 x$

3. $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

4. $\dfrac{\sin t - 1}{\cos t} = \dfrac{\cos t - \cot t}{\cos t \cot t}$

5. $\dfrac{1 - \cos 2x}{\sin 2x} = \tan x$

6. $\sin\theta\cos\theta + \cos^2\theta = \dfrac{\cos\theta(1 + \cot\theta)}{\csc\theta}$

7. $\dfrac{\sin x}{\tan x} + \dfrac{\cos x}{\cot x} = \sin x + \cos x$

8. $\sin^2\frac{t}{2} = \dfrac{\tan t - \sin t}{2\tan t}$

9. $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

10. $\dfrac{1 + \csc x}{\sec x} - \cot x = \cos x$

11. $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1 - \tan x}{1 + \tan x}$

12. $2\sin^3\theta\cos\theta + 2\sin\theta\cos^3\theta = \sin 2\theta$

13. $\dfrac{\sin t + \cos t}{\sec t + \csc t} = \dfrac{\sin t}{\sec t}$

14. $\sec 2x = \dfrac{\sec^2 x}{2 - \sec^2 x}$

15. $\tan(\alpha+\beta)\tan(\alpha-\beta) = \dfrac{\tan^2\alpha - \tan^2\beta}{1 - \tan^2\alpha\tan^2\beta}$

16. $\csc\theta + \cot\theta = \dfrac{\sin\theta}{1 - \cos\theta}$

17. $\dfrac{1}{\csc 2x} = \dfrac{2\tan x}{1 + \tan^2 x}$

18. $\dfrac{\sec t - 1}{t\sec t} = \dfrac{1 - \cos t}{t}$

Use the following conditions to solve Exercises 19–22:

$$\sin\alpha = \frac{3}{5}, \quad \frac{\pi}{2} < \alpha < \pi$$
$$\cos\beta = -\frac{12}{13}, \quad \pi < \beta < \frac{3\pi}{2}.$$

Find the exact value of each of the following.

19. $\cos(\alpha - \beta)$

20. $\tan(\alpha + \beta)$

21. $\sin 2\alpha$

22. $\cos\frac{\beta}{2}$

In Exercises 23–26, find the exact value of each expression. Do not use a calculator.

23. $\sin\left(\frac{3\pi}{4} + \frac{5\pi}{6}\right)$

24. $\cos^2 15° - \sin^2 15°$

25. $\cos\frac{5\pi}{12}\cos\frac{\pi}{12} + \sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

26. $\tan 22.5°$