93. a. Make a sketch showing that $\cot 2\theta = -\frac{7}{24}$ for $90° < 2\theta < 180°$.

b. Use your sketch from part (a) to determine the value of $\cos 2\theta$.

c. Use the value of $\cos 2\theta$ from part (b) and the identities

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} \quad \text{and} \quad \cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}}$$

to determine the values of $\sin\theta$ and $\cos\theta$.

d. In part (c), why did we not write $\pm$ before the radical in each formula?

94. The equation $3x^2 - 2\sqrt{3}xy + y^2 + 2x + 2\sqrt{3}y = 0$ is in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. Use the equation to determine the value of $B^2 - 4AC$.

Chapter 9 Mid-Chapter Check Point

What You Know: We learned that the four conic sections are the circle, the ellipse, the hyperbola, and the parabola. Prior to this chapter, we graphed circles with center (h, k) and radius r:

$$(x-h)^2 + (y-k)^2 = r^2.$$

In this chapter, you learned to graph ellipses centered at the origin and ellipses centered at (h, k):

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad \text{or} \quad \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1,\ a^2 > b^2.$$

We saw that the larger denominator (a^2) determines whether the major axis is horizontal or vertical. We used vertices and asymptotes to graph hyperbolas centered at the origin and hyperbolas centered at (h, k):

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \text{ or } \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1.$$

We used $c^2 = a^2 - b^2$ to locate the foci of an ellipse. We used $c^2 = a^2 + b^2$ to locate the foci of a hyperbola. Finally, we used the vertex and the latus rectum to graph parabolas with vertices at the origin and parabolas with vertices at (h, k):

$$(y-k)^2 = 4p(x-h) \quad \text{or} \quad (x-h)^2 = 4p(y-k).$$

In Exercises 1–5, graph each ellipse. Give the location of the foci.

1. $\frac{x^2}{25} + \frac{y^2}{4} = 1$

2. $9x^2 + 4y^2 = 36$

3. $\frac{(x-2)^2}{16} + \frac{(y+1)^2}{25} = 1$

4. $\frac{(x+2)^2}{25} + \frac{(y-1)^2}{16} = 1$

5. $x^2 + 9y^2 - 4x + 54y + 49 = 0$

In Exercises 6–11, graph each hyperbola. Give the location of the foci and the equations of the asymptotes.

6. $\frac{x^2}{9} - y^2 = 1$

7. $\frac{y^2}{9} - x^2 = 1$

8. $y^2 - 4x^2 = 16$

9. $4x^2 - 49y^2 = 196$

10. $\frac{(x-2)^2}{9} - \frac{(y+2)^2}{16} = 1$

11. $4x^2 - y^2 + 8x + 6y + 11 = 0$

In Exercises 12–13, graph each parabola. Give the location of the focus and the directrix.

12. $(x-2)^2 = -12(y+1)$

13. $y^2 - 2x - 2y - 5 = 0$

In Exercises 14–21, graph each equation.

14. $x^2 + y^2 = 4$

15. $x + y = 4$

16. $x^2 - y^2 = 4$

17. $x^2 + 4y^2 = 4$

18. $(x+1)^2 + (y-1)^2 = 4$

19. $x^2 + 4(y-1)^2 = 4$

20. $(x-1)^2 - (y-1)^2 = 4$

21. $(y+1)^2 = 4(x-1)$

In Exercises 22–27, find the standard form of the equation of the conic section satisfying the given conditions.

22. Ellipse; Foci: $(-4, 0), (4, 0)$; Vertices: $(-5, 0), (5, 0)$

23. Ellipse; Endpoints of major axis: $(-8, 2), (10, 2)$; Foci: $(-4, 2), (6, 2)$

24. Hyperbola; Foci: $(0, -3), (0, 3)$; Vertices: $(0, -2), (0, 2)$

25. Hyperbola; Foci: $(-4, 5), (2, 5)$; Vertices: $(-3, 5), (1, 5)$

26. Parabola; Focus: $(4, 5)$; Directrix: $y = -1$

27. Parabola; Focus: $(-2, 6)$; Directrix: $x = 8$

28. A semielliptical archway over a one-way road has a height of 10 feet and a width of 30 feet. A truck has a width of 10 feet and a height of 9.5 feet. Will this truck clear the opening of the archway?

29. A lithotriper is used to disentegrate kidney stones. The patient is placed within an elliptical device with the kidney centered at one focus, while ultrasound waves from the other focus hit the walls and are reflected to the kidney stone, shattering the stone. Suppose that the length of the major axis of the ellipse is 40 centimeters and the length of the minor axis is 20 centimeters. How far from the kidney stone should the electrode that sends the ultrasound waves be placed in order to shatter the stone?

30. An explosion is recorded by two forest rangers, one at a primary station and the other at an outpost 6 kilometers away. The ranger at the primary station hears the explosion 6 seconds before the ranger at the outpost.

a. Assuming sound travels at 0.35 kilometer per second, write an equation in standard form that gives all the possible locations of the explosion. Use a coordinate system with the two ranger stations on the x-axis and the midpoint between the stations at the origin.

b. Graph the equation that gives the possible locations of the explosion. Show the locations of the ranger stations in your drawing.

31. A domed ceiling is a parabolic surface. Ten meters down from the top of the dome, the ceiling is 15 meters wide. For the best lighting on the floor, a light source should be placed at the focus of the parabolic surface. How far from the top of the dome, to the nearest tenth of a meter, should the light source be placed?