Prerequisites: Fundamental Concepts of Algebra

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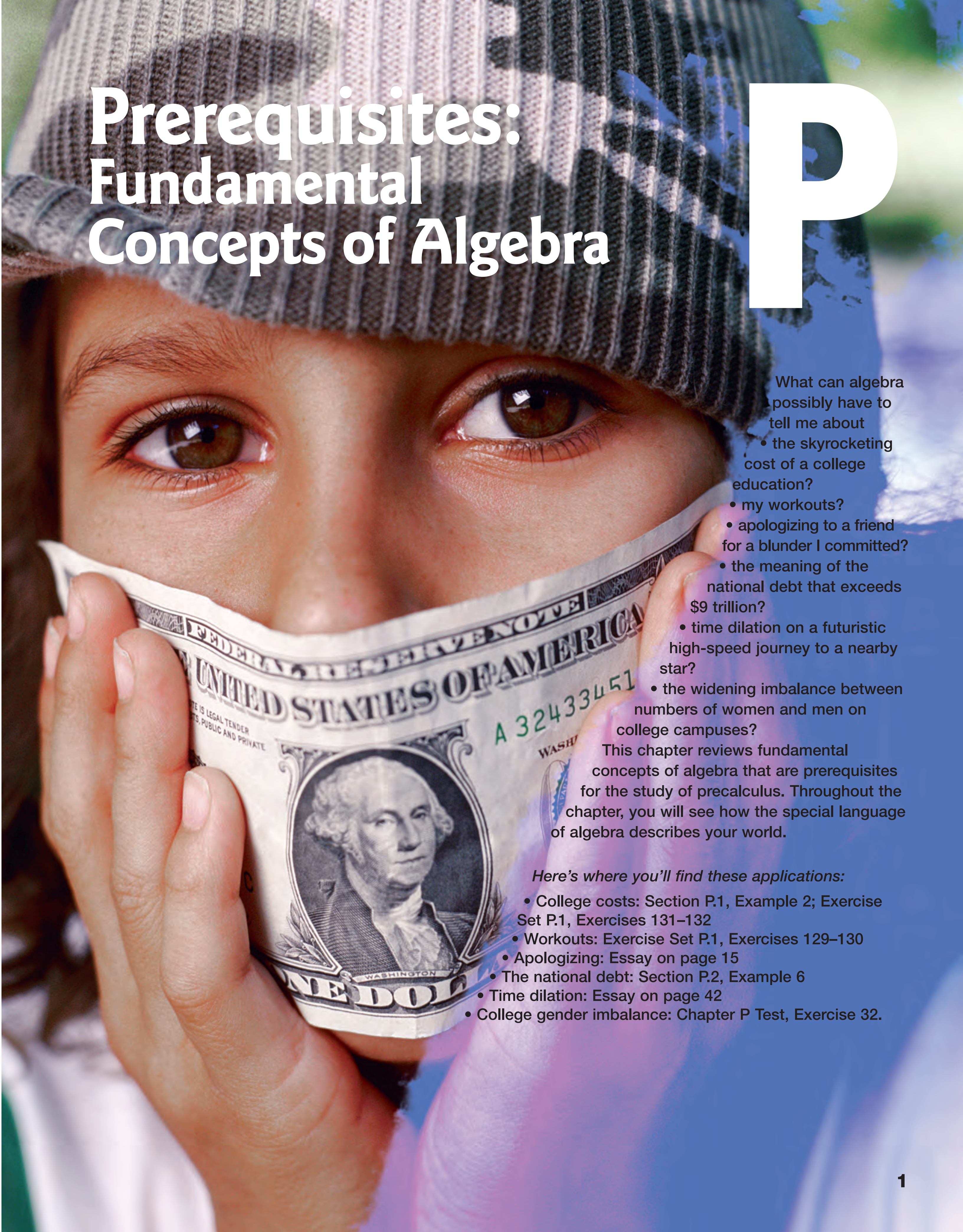


What can algebra possibly have to tell me about

- the skyrocketing cost of a college education?
- my workouts?
- apologizing to a friend for a blunder I committed?
- the meaning of the national debt that exceeds \$9 trillion?
- time dilation on a futuristic high-speed journey to a nearby star?
- the widening imbalance between numbers of women and men on college campuses?

This chapter reviews fundamental concepts of algebra that are prerequisites for the study of precalculus. Throughout the chapter, you will see how the special language of algebra describes your world.

Here's where you'll find these applications:

- College costs: Section P.1, Example 2; Exercise Set P.1, Exercises 131–132
- Workouts: Exercise Set P.1, Exercises 129–130
- Apologizing: Essay on page 15
- The national debt: Section P.2, Example 6
- Time dilation: Essay on page 42
- College gender imbalance: Chapter P Test, Exercise 32.

Section P.1

Algebraic Expressions, Mathematical Models, and Real Numbers

Objectives

1. Evaluate algebraic expressions.
2. Use mathematical models.
3. Find the intersection of two sets.
4. Find the union of two sets.
5. Recognize subsets of the real numbers.
6. Use inequality symbols.
7. Evaluate absolute value.
8. Use absolute value to express distance.
9. Identify properties of the real numbers.
10. Simplify algebraic expressions.

How would your lifestyle change if a gallon of gas cost \$9.15? Or if the price of a staple such as milk was \$15? That's how much those products would cost if their prices had increased at the same rate college tuition has increased since 1980. (*Source*: Center for College Affordability and Productivity) In this section, you will learn how the special language of algebra describes your world, including the skyrocketing cost of a college education.

Algebraic Expressions

Algebra uses letters, such as x and y, to represent numbers. If a letter is used to represent various numbers, it is called a **variable**. For example, imagine that you are basking in the sun on the beach. We can let x represent the number of minutes that you can stay in the sun without burning with no sunscreen. With a number 6 sunscreen, exposure time without burning is six times as long, or 6 times x. This can be written $6 \cdot x$, but it is usually expressed as $6x$. Placing a number and a letter next to one another indicates multiplication.

Notice that $6x$ combines the number 6 and the variable x using the operation of multiplication. A combination of variables and numbers using the operations of addition, subtraction, multiplication, or division, as well as powers or roots, is called an **algebraic expression**. Here are some examples of algebraic expressions:

$$x + 6, \quad x - 6, \quad 6x, \quad \frac{x}{6}, \quad 3x + 5, \quad x^2 - 3, \quad \sqrt{x} + 7.$$

Many algebraic expressions involve *exponents*. For example, the algebraic expression

$$17x^2 + 261x + 3257$$

approximates the average cost of tuition and fees at public U.S. colleges for the school year ending x years after 2000. The expression x^2 means $x \cdot x$, and is read "x to the second power" or "x squared." The exponent, 2, indicates that the base, x, appears as a factor two times.

Exponential Notation

If n is a counting number (1, 2, 3, and so on),

Exponent or Power

$$b^n = \underbrace{b \cdot b \cdot b \cdot \cdots \cdot b}_{b \text{ appears as a factor } n \text{ times.}}$$

Base

b^n is read "the nth power of b" or "b to the nth power." Thus, the nth power of b is defined as the product of n factors of b. The expression b^n is called an **exponential expression**. Furthermore, $b^1 = b$.

For example,

$$8^2 = 8 \cdot 8 = 64, \quad 5^3 = 5 \cdot 5 \cdot 5 = 125, \quad \text{and} \quad 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16.$$

1 Evaluate algebraic expressions.

Evaluating Algebraic Expressions

Evaluating an algebraic expression means to find the value of the expression for a given value of the variable.

Many algebraic expressions involve more than one operation. Evaluating an algebraic expression without a calculator involves carefully applying the following order of operations agreement:

The Order of Operations Agreement

1. Perform operations within the innermost parentheses and work outward. If the algebraic expression involves a fraction, treat the numerator and the denominator as if they were each enclosed in parentheses.
2. Evaluate all exponential expressions.
3. Perform multiplications and divisions as they occur, working from left to right.
4. Perform additions and subtractions as they occur, working from left to right.

EXAMPLE 1 Evaluating an Algebraic Expression

Evaluate $7 + 5(x - 4)^3$ for $x = 6$.

Solution

$7 + 5(x - 4)^3 = 7 + 5(6 - 4)^3$ Replace x with 6.

$= 7 + 5(2)^3$ First work inside parentheses: $6 - 4 = 2$.

$= 7 + 5(8)$ Evaluate the exponential expression: $2^3 = 2 \cdot 2 \cdot 2 = 8$.

$= 7 + 40$ Multiply: $5(8) = 40$.

$= 47$ Add.

Check Point 1 Evaluate $8 + 6(x - 3)^2$ for $x = 13$.

2 Use mathematical models.

Formulas and Mathematical Models

An **equation** is formed when an equal sign is placed between two algebraic expressions. One aim of algebra is to provide a compact, symbolic description of the world. These descriptions involve the use of *formulas*. A **formula** is an equation that uses variables to express a relationship between two or more quantities.

Here are two examples of formulas related to heart rate and exercise.

Couch-Potato Exercise

$$H = \frac{1}{5}(220 - a)$$

Heart rate, in beats per minute, | is | $\frac{1}{5}$ of | the difference between 220 and your age.

Working It

$$H = \frac{9}{10}(220 - a)$$

Heart rate, in beats per minute, | is | $\frac{9}{10}$ of | the difference between 220 and your age.

The process of finding formulas to describe real-world phenomena is called **mathematical modeling**. Such formulas, together with the meaning assigned to the variables, are called **mathematical models**. We often say that these formulas model, or describe, the relationships among the variables.

EXAMPLE 2 Modeling the Cost of Attending a Public College

The bar graph in **Figure P.1** shows the average cost of tuition and fees for public four-year colleges, adjusted for inflation. The formula

$$T = 17x^2 + 261x + 3257$$

models the average cost of tuition and fees, T, for public U.S. colleges for the school year ending x years after 2000.

a. Use the formula to find the average cost of tuition and fees at public U.S. colleges for the school year ending in 2007.

b. By how much does the formula underestimate or overestimate the actual cost shown in **Figure P.1**?

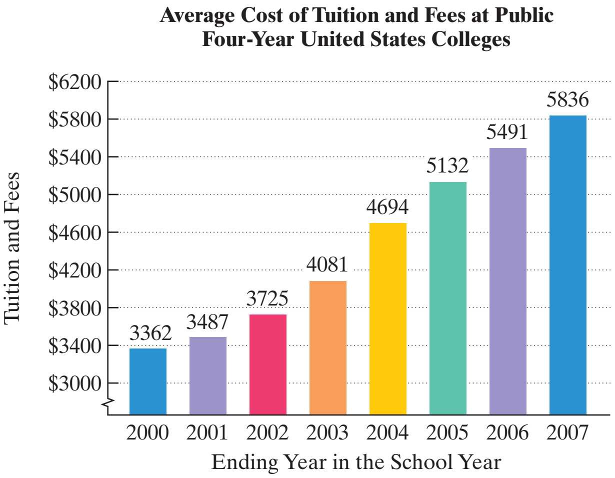


Figure P.1
Source: The College Board

Solution

a. Because 2007 is 7 years after 2000, we substitute 7 for x in the given formula. Then we use the order of operations to find T, the average cost of tuition and fees for the school year ending in 2007.

$T = 17x^2 + 261x + 3257$	This is the given mathematical model.
$T = 17(7)^2 + 261(7) + 3257$	Replace each occurrence of x with 7.
$T = 17(49) + 261(7) + 3257$	Evaluate the exponential expression $7^2 = 7 \cdot 7 = 49$.
$T = 833 + 1827 + 3257$	Multiply from left to right: $17(49) = 833$ and $261(7) = 1827$.
$T = 5917$	Add.

The formula indicates that for the school year ending in 2007, the average cost of tuition and fees at public U.S. colleges was $5917.

b. **Figure P.1** shows that the average cost of tuition and fees for the school year ending in 2007 was $5836.

The cost obtained from the formula, $5917, overestimates the actual data value by $5917 − $5836, or by $81.

Check Point 2 Assuming trends indicated by the data in **Figure P.1** continue, use the formula $T = 17x^2 + 261x + 3257$, described in Example 2, to project the average cost of tuition and fees at public U.S. colleges for the school year ending in 2010.

Sometimes a mathematical model gives an estimate that is not a good approximation or is extended to include values of the variable that do not make sense. In these cases, we say that **model breakdown** has occurred. For example, it is not likely that the formula in Example 2 would give a good estimate of tuition and fees in 2050 because it is too far in the future. Thus, model breakdown would occur.

Sets

Before we describe the set of real numbers, let's be sure you are familiar with some basic ideas about sets. A **set** is a collection of objects whose contents can be clearly determined. The objects in a set are called the **elements** of the set. For example, the set of numbers used for counting can be represented by

$$\{1, 2, 3, 4, 5, \ldots\}.$$

The braces, { }, indicate that we are representing a set. This form of representation, called the **roster method**, uses commas to separate the elements of the set. The symbol consisting of three dots after the 5, called an *ellipsis*, indicates that there is no final element and that the listing goes on forever.

Study Tip

Grouping symbols such as parentheses, ( ), and square brackets, [ ], are not used to represent sets. Only commas are used to separate the elements of a set. Separators such as colons or semicolons are not used.

A set can also be written in **set-builder notation**. In this notation, the elements of the set are described, but not listed. Here is an example:

$$\{x \mid x \text{ is a counting number less than } 6\}.$$

The set of all x | such that | x is a counting number less than 6.

The same set written using the roster method is

$$\{1, 2, 3, 4, 5\}.$$

3 Find the intersection of two sets.

If A and B are sets, we can form a new set consisting of all elements that are in both A and B. This set is called the *intersection* of the two sets.

Definition of the Intersection of Sets

The **intersection** of sets A and B, written $A \cap B$, is the set of elements common to both set A **and** set B. This definition can be expressed in set-builder notation as follows:

$$A \cap B = \{x \mid x \text{ is an element of } A \text{ AND } x \text{ is an element of } B\}.$$

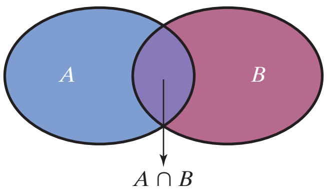


Figure P.2 Picturing the intersection of two sets

Figure P.2 shows a useful way of picturing the intersection of sets A and B. The figure indicates that $A \cap B$ contains those elements that belong to both A and B at the same time.

EXAMPLE 3 Finding the Intersection of Two Sets

Find the intersection: $\{7, 8, 9, 10, 11\} \cap \{6, 8, 10, 12\}$.

Solution The elements common to $\{7, 8, 9, 10, 11\}$ and $\{6, 8, 10, 12\}$ are 8 and 10. Thus,

$$\{7, 8, 9, 10, 11\} \cap \{6, 8, 10, 12\} = \{8, 10\}.$$

Check Point 3 Find the intersection: $\{3, 4, 5, 6, 7\} \cap \{3, 7, 8, 9\}$.

If a set has no elements, it is called the **empty set**, or the **null set**, and is represented by the symbol Ø (the Greek letter phi). Here is an example that shows how the empty set can result when finding the intersection of two sets:

$$\{2, 4, 6\} \cap \{3, 5, 7\} = \varnothing.$$

These sets have no common elements.

Their intersection has no elements and is the empty set.

4 Find the union of two sets.

Another set that we can form from sets A and B consists of elements that are in A or B or in both sets. This set is called the *union* of the two sets.

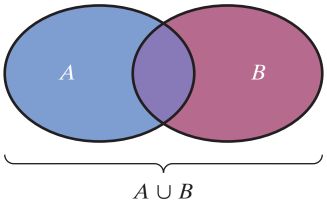


Figure P.3 Picturing the union of two sets

Definition of the Union of Sets

The **union** of sets A and B, written $A \cup B$, is the set of elements that are members of set A **or** of set B or of both sets. This definition can be expressed in set-builder notation as follows:

$$A \cup B = \{x \mid x \text{ is an element of } A \text{ OR } x \text{ is an element of } B\}.$$

Figure P.3 shows a useful way of picturing the union of sets A and B. The figure indicates that $A \cup B$ is formed by joining the sets together.

We can find the union of set A and set B by listing the elements of set A. Then, we include any elements of set B that have not already been listed. Enclose all elements that are listed with braces. This shows that the union of two sets is also a set.

Study Tip

When finding the union of two sets, do not list twice any elements that appear in both sets.

EXAMPLE 4 Finding the Union of Two Sets

Find the union: $\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\}$.

Solution To find $\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\}$, start by listing all the elements from the first set, namely 7, 8, 9, 10, and 11. Now list all the elements from the second set that are not in the first set, namely 6 and 12. The union is the set consisting of all these elements. Thus,

$$\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\} = \{6, 7, 8, 9, 10, 11, 12\}.$$

Check Point 4 Find the union: $\{3, 4, 5, 6, 7\} \cup \{3, 7, 8, 9\}$.

5 Recognize subsets of the real numbers.

The Set of Real Numbers

The sets that make up the real numbers are summarized in **Table P.1**. We refer to these sets as **subsets** of the real numbers, meaning that all elements in each subset are also elements in the set of real numbers.

Notice the use of the symbol $\approx$ in the examples of irrational numbers. The symbol means "is approximately equal to." Thus,

$$\sqrt{2} \approx 1.414214.$$

We can verify that this is only an approximation by multiplying 1.414214 by itself. The product is very close to, but not exactly, 2:

$$1.414214 \times 1.414214 = 2.000001237796.$$

Technology

A calculator with a square root key gives a decimal approximation for $\sqrt{2}$, not the exact value.

Table P.1 Important Subsets of the Real Numbers

Name	Description	Examples
Natural numbers $\mathbb{N}$	$\{1, 2, 3, 4, 5, \ldots\}$ These are the numbers that we use for counting.	2, 3, 5, 17
Whole numbers $\mathbb{W}$	$\{0, 1, 2, 3, 4, 5, \ldots\}$ The set of whole numbers includes 0 and the natural numbers.	0, 2, 3, 5, 17
Integers $\mathbb{Z}$	$\{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$ The set of integers includes the negatives of the natural numbers and the whole numbers.	$-17, -5, -3, -2, 0, 2, 3, 5, 17$
Rational numbers $\mathbb{Q}$	$\left\{\frac{a}{b} \mid a \text{ and } b \text{ are integers and } b \neq 0\right\}$ (This means that b is not equal to zero.) The set of rational numbers is the set of all numbers that can be expressed as a quotient of two integers, with the denominator not 0. Rational numbers can be expressed as terminating or repeating decimals.	$-17 = \frac{-17}{1}, -5 = \frac{-5}{1}, -3, -2,$ $0, 2, 3, 5, 17,$ $\frac{2}{5} = 0.4,$ $\frac{-2}{3} = -0.6666\ldots = -0.\overline{6}$
Irrational numbers $\mathbb{I}$	The set of irrational numbers is the set of all numbers whose decimal representations are neither terminating nor repeating. Irrational numbers cannot be expressed as a quotient of integers.	$\sqrt{2} \approx 1.414214$ $-\sqrt{3} \approx -1.73205$ $\pi \approx 3.142$ $-\frac{\pi}{2} \approx -1.571$

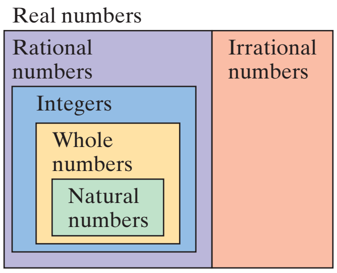


Figure P.4 Every real number is either rational or irrational.

Not all square roots are irrational. For example, $\sqrt{25} = 5$ because $5^2 = 5 \cdot 5 = 25$. Thus, $\sqrt{25}$ is a natural number, a whole number, an integer, and a rational number $\left(\sqrt{25} = \frac{5}{1}\right)$.

The set of *real numbers* is formed by taking the union of the sets of rational numbers and irrational numbers. Thus, every real number is either rational or irrational, as shown in **Figure P.4**.

Real Numbers

The set of **real numbers** is the set of numbers that are either rational or irrational:

$$\{x \mid x \text{ is rational or } x \text{ is irrational}\}.$$

The symbol $\mathbb{R}$ is used to represent the set of real numbers. Thus,

$$\mathbb{R} = \{x \mid x \text{ is rational}\} \cup \{x \mid x \text{ is irrational}\}.$$

EXAMPLE 5 Recognizing Subsets of the Real Numbers

Consider the following set of numbers:

$$\left\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\right\}.$$

List the numbers in the set that are

a. natural numbers. **b.** whole numbers. **c.** integers.

d. rational numbers. **e.** irrational numbers. **f.** real numbers.

Solution

a. Natural numbers: The natural numbers are the numbers used for counting. The only natural number in the set $\left\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\right\}$ is $\sqrt{81}$ because $\sqrt{81} = 9$. (9 multiplied by itself, or 9^2, is 81.)

b. Whole numbers: The whole numbers consist of the natural numbers and 0. The elements of the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that are whole numbers are 0 and $\sqrt{81}$.

c. Integers: The integers consist of the natural numbers, 0, and the negatives of the natural numbers. The elements of the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that are integers are $\sqrt{81}$, 0, and -7.

d. Rational numbers: All numbers in the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that can be expressed as the quotient of integers are rational numbers. These include $-7\left(-7 = \frac{-7}{1}\right)$, $-\frac{3}{4}$, $0\left(0 = \frac{0}{1}\right)$, and $\sqrt{81}\left(\sqrt{81} = \frac{9}{1}\right)$. Furthermore, all numbers in the set that are terminating or repeating decimals are also rational numbers. These include $0.\overline{6}$ and 7.3.

e. Irrational numbers: The irrational numbers in the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ are $\sqrt{5}\left(\sqrt{5} \approx 2.236\right)$ and $\pi(\pi \approx 3.14)$. Both $\sqrt{5}$ and π are only approximately equal to 2.236 and 3.14, respectively. In decimal form, $\sqrt{5}$ and π neither terminate nor have blocks of repeating digits.

f. Real numbers: All the numbers in the given set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ are real numbers.

Check Point 5 Consider the following set of numbers:

$$\left\{-9, -1.3, 0, 0.\overline{3}, \frac{\pi}{2}, \sqrt{9}, \sqrt{10}\right\}.$$

List the numbers in the set that are

a. natural numbers. **b.** whole numbers. **c.** integers.
d. rational numbers. **e.** irrational numbers. **f.** real numbers.

The Real Number Line

The **real number line** is a graph used to represent the set of real numbers. An arbitrary point, called the **origin**, is labeled 0. Select a point to the right of 0 and label it 1. The distance from 0 to 1 is called the **unit distance**. Numbers to the right of the origin are **positive** and numbers to the left of the origin are **negative**. The real number line is shown in **Figure P.5**.

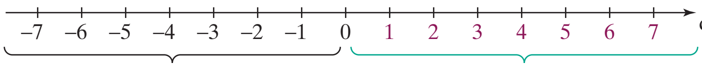
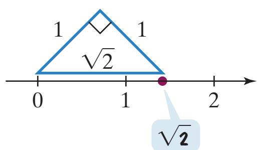


Figure P.5 The real number line

Study Tip

Wondering how we located $\sqrt{2}$ as a precise point on the number line in **Figure P.6**? We used a right triangle with both legs of length 1. The remaining side measures $\sqrt{2}$.



We'll have lots more to say about right triangles later in the chapter.

Real numbers are **graphed** on a number line by placing a dot at the correct location for each number. The integers are easiest to locate. In **Figure P.6**, we've graphed six rational numbers and three irrational numbers on a real number line.

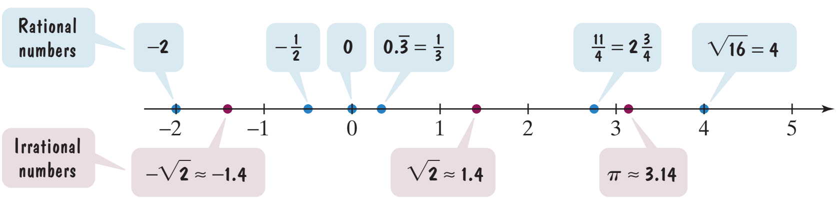


Figure P.6 Graphing numbers on a real number line

Every real number corresponds to a point on the number line and every point on the number line corresponds to a real number. We say that there is a **one-to-one correspondence** between all the real numbers and all points on a real number line.

6 Use inequality symbols.

Ordering the Real Numbers

On the real number line, the real numbers increase from left to right. The lesser of two real numbers is the one farther to the left on a number line. The greater of two real numbers is the one farther to the right on a number line.

Look at the number line in **Figure P.7**. The integers -4 and -1 are graphed.

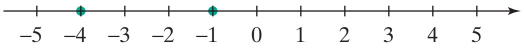


Figure P.7

Observe that -4 is to the left of -1 on the number line. This means that -4 is less than -1.

$$-4 < -1$$

-4 is less than -1 because -4 is to the **left** of -1 on the number line.

In **Figure P.7**, we can also observe that -1 is to the right of -4 on the number line. This means that -1 is greater than -4.

$$-1 > -4$$

-1 is greater than -4 because -1 is to the **right** of -4 on the number line.

The symbols $<$ and $>$ are called **inequality symbols**. These symbols always point to the lesser of the two real numbers when the inequality statement is true.

-4 is less than -1. $\quad -4 < -1 \quad$ The symbol points to -4, the lesser number.

-1 is greater than -4. $\quad -1 > -4 \quad$ The symbol still points to -4, the lesser number.

The symbols $<$ and $>$ may be combined with an equal sign, as shown in the following table:

This inequality is true if either the $<$ part or the $=$ part is true.

This inequality is true if either the $>$ part or the $=$ part is true.

Symbols	Meaning	Examples	Explanation
$a \le b$	a is less than or equal to b.	$2 \le 9$ $9 \le 9$	Because $2 < 9$ Because $9 = 9$
$b \ge a$	b is greater than or equal to a.	$9 \ge 2$ $2 \ge 2$	Because $9 > 2$ Because $2 = 2$

7 Evaluate absolute value.

Absolute Value

The **absolute value** of a real number a, denoted by $|a|$, is the distance from 0 to a on the number line. This distance is always taken to be nonnegative. For example, the real number line in **Figure P.8** shows that

$$|-3| = 3 \quad \text{and} \quad |5| = 5.$$

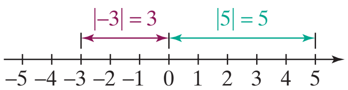


Figure P.8 Absolute value as the distance from 0

The absolute value of -3 is 3 because -3 is 3 units from 0 on the number line. The absolute value of 5 is 5 because 5 is 5 units from 0 on the number line. The absolute value of a positive real number or 0 is the number itself. The absolute value of a negative real number, such as -3, is the number without the negative sign.

We can define the absolute value of the real number x without referring to a number line. The algebraic definition of the absolute value of x is given as follows:

Definition of Absolute Value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

If x is nonnegative (that is, $x \geq 0$), the absolute value of x is the number itself. For example,

$$|5| = 5 \qquad |\pi| = \pi \qquad \left|\frac{1}{3}\right| = \frac{1}{3} \qquad |0| = 0.$$

Zero is the only number whose absolute value is 0.

If x is a negative number (that is, $x < 0$), the absolute value of x is the opposite of x. This makes the absolute value positive. For example,

$$|-3| = -(-3) = 3 \qquad |-\pi| = -(-\pi) = \pi \qquad \left|-\frac{1}{3}\right| = -\left(-\frac{1}{3}\right) = \frac{1}{3}.$$

This middle step is usually omitted.

EXAMPLE 6 Evaluating Absolute Value

Rewrite each expression without absolute value bars:

a. $|\sqrt{3} - 1|$ **b.** $|2 - \pi|$ **c.** $\frac{|x|}{x}$ if $x < 0$.

Solution

a. Because $\sqrt{3} \approx 1.7$, the number inside the absolute value bars, $\sqrt{3} - 1$, is positive. The absolute value of a positive number is the number itself. Thus,

$$|\sqrt{3} - 1| = \sqrt{3} - 1.$$

b. Because $\pi \approx 3.14$, the number inside the absolute value bars, $2 - \pi$, is negative. The absolute value of x when $x < 0$ is $-x$. Thus,

$$|2 - \pi| = -(2 - \pi) = \pi - 2.$$

c. If $x < 0$, then $|x| = -x$. Thus,

$$\frac{|x|}{x} = \frac{-x}{x} = -1.$$

Check Point 6 Rewrite each expression without absolute value bars:

a. $|1 - \sqrt{2}|$ **b.** $|\pi - 3|$ **c.** $\frac{|x|}{x}$ if $x > 0$.

Listed below are several basic properties of absolute value. Each of these properties can be derived from the definition of absolute value.

Discovery

Verify the triangle inequality if $a = 4$ and $b = 5$. Verify the triangle inequality if $a = 4$ and $b = -5$.

When does equality occur in the triangle inequality and when does inequality occur? Verify your observation with additional number pairs.

Properties of Absolute Value

For all real numbers a and b,

1. $|a| \geq 0$ **2.** $|-a| = |a|$ **3.** $a \leq |a|$

4. $|ab| = |a||b|$ **5.** $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, \quad b \neq 0$

6. $|a + b| \leq |a| + |b|$ (called the triangle inequality)

8 Use absolute value to express distance.

Distance between Points on a Real Number Line

Absolute value is used to find the distance between two points on a real number line. If a and b are any real numbers, the **distance between *a* and *b*** is the absolute value of their difference. For example, the distance between 4 and 10 is 6. Using absolute value, we find this distance in one of two ways:

$$|10 - 4| = |6| = 6 \quad \text{or} \quad |4 - 10| = |-6| = 6.$$

The distance between 4 and 10 on the real number line is 6.

Notice that we obtain the same distance regardless of the order in which we subtract.

Distance between Two Points on the Real Number Line

If a and b are any two points on a real number line, then the distance between a and b is given by

$$|a - b| \quad \text{or} \quad |b - a|.$$

EXAMPLE 7 Distance between Two Points on a Number Line

Find the distance between -5 and 3 on the real number line.

Solution Because the distance between a and b is given by $|a - b|$, the distance between -5 and 3 is

$$|-5 - 3| = |-8| = 8.$$

$a = -5$ $b = 3$

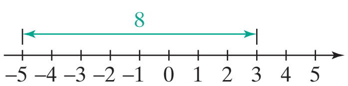


Figure P.9 The distance between -5 and 3 is 8.

Figure P.9 verifies that there are 8 units between -5 and 3 on the real number line. We obtain the same distance if we reverse the order of the subtraction:

$$|3 - (-5)| = |8| = 8.$$

Check Point 7 Find the distance between -4 and 5 on the real number line.

9 Identify properties of the real numbers.

Properties of Real Numbers and Algebraic Expressions

When you use your calculator to add two real numbers, you can enter them in any order. The fact that two real numbers can be added in any order is called the **commutative property of addition**. You probably use this property, as well as other properties of real numbers listed in **Table P.2**, without giving it much thought. The properties of the real numbers are especially useful when working with algebraic expressions. For each property listed in **Table P.2**, a, b, and c represent real numbers, variables, or algebraic expressions.

Table P.2 Properties of the Real Numbers

Name	Meaning	Examples
Commutative Property of Addition	Changing order when adding does not affect the sum. $a + b = b + a$	• $13 + 7 = 7 + 13$ • $13x + 7 = 7 + 13x$
Commutative Property of Multiplication	Changing order when multiplying does not affect the product. $ab = ba$	• $\sqrt{2}\cdot\sqrt{5} = \sqrt{5}\cdot\sqrt{2}$ • $x\cdot 6 = 6x$
Associative Property of Addition	Changing grouping when adding does not affect the sum. $(a + b) + c = a + (b + c)$	• $3 + (8 + x) = (3 + 8) + x = 11 + x$
Associative Property of Multiplication	Changing grouping when multiplying does not affect the product. $(ab)c = a(bc)$	• $-2(3x) = (-2\cdot 3)x = -6x$
Distributive Property of Multiplication over Addition	Multiplication distributes over addition. $a\cdot(b + c) = a\cdot b + a\cdot c$	• $7(4 + \sqrt{3}) = 7\cdot 4 + 7\cdot\sqrt{3} = 28 + 7\sqrt{3}$ • $5(3x + 7) = 5\cdot 3x + 5\cdot 7 = 15x + 35$
Identity Property of Addition	Zero can be deleted from a sum. $a + 0 = a$ $0 + a = a$	• $\sqrt{3} + 0 = \sqrt{3}$ • $0 + 6x = 6x$
Identity Property of Multiplication	One can be deleted from a product. $a\cdot 1 = a$ $1\cdot a = a$	• $1\cdot\pi = \pi$ • $13x\cdot 1 = 13x$
Inverse Property of Addition	The sum of a real number and its additive inverse gives 0, the additive identity. $a + (-a) = 0$ $(-a) + a = 0$	• $\sqrt{5} + (-\sqrt{5}) = 0$ • $-\pi + \pi = 0$ • $6x + (-6x) = 0$ • $(-4y) + 4y = 0$
Inverse Property of Multiplication	The product of a nonzero real number and its multiplicative inverse gives 1, the multiplicative identity. $a\cdot\frac{1}{a} = 1,\quad a \neq 0$ $\frac{1}{a}\cdot a = 1,\quad a \neq 0$	• $7\cdot\frac{1}{7} = 1$ • $\left(\frac{1}{x-3}\right)(x - 3) = 1,\quad x \neq 3$

The Associative Property and the English Language

In the English language, phrases can take on different meanings depending on the way the words are associated with commas.

Here are three examples.

- Woman, without her man, is nothing.
  Woman, without her, man is nothing.
- What's the latest dope?
  What's the latest, dope?
- Population of Amsterdam broken down by age and sex
  Population of Amsterdam, broken down by age and sex

The properties of the real numbers in **Table P.2** apply to the operations of addition and multiplication. Subtraction and division are defined in terms of addition and multiplication.

Definitions of Subtraction and Division

Let a and b represent real numbers.

Subtraction: $a - b = a + (-b)$
We call $-b$ the **additive inverse** or **opposite** of b.

Division: $a \div b = a \cdot \frac{1}{b}$, where $b \neq 0$
We call $\frac{1}{b}$ the **multiplicative inverse** or **reciprocal** of b. The quotient of a and b, $a \div b$, can be written in the form $\frac{a}{b}$, where a is the **numerator** and b the **denominator** of the fraction.

Because subtraction is defined in terms of adding an inverse, the distributive property can be applied to subtraction:

$$a(b - c) = ab - ac$$
$$(b - c)a = ba - ca.$$

For example,

$$4(2x - 5) = 4 \cdot 2x - 4 \cdot 5 = 8x - 20.$$

10 Simplify algebraic expressions.

Simplifying Algebraic Expressions

The **terms** of an algebraic expression are those parts that are separated by addition. For example, consider the algebraic expression

$$7x - 9y + z - 3,$$

which can be expressed as

$$7x + (-9y) + z + (-3).$$

This expression contains four terms, namely $7x$, $-9y$, z, and -3.

The numerical part of a term is called its **coefficient**. In the term $7x$, the 7 is the coefficient. If a term containing one or more variables is written without a coefficient, the coefficient is understood to be 1. Thus, z means $1z$. If a term is a constant, its coefficient is that constant. Thus, the coefficient of the constant term -3 is -3.

$$7x + (-9y) + z + (-3)$$

Coefficient is 7.
Coefficient is −9.
Coefficient is 1; z means $1z$.
Coefficient is −3.

The parts of each term that are multiplied are called the **factors** of the term. The factors of the term $7x$ are 7 and x.

Like terms are terms that have exactly the same variable factors. For example, $3x$ and $7x$ are like terms. The distributive property in the form

$$ba + ca = (b + c)a$$

enables us to add or subtract like terms. For example,

$$3x + 7x = (3 + 7)x = 10x$$
$$7y^2 - y^2 = 7y^2 - 1y^2 = (7 - 1)y^2 = 6y^2.$$

This process is called **combining like terms**.

An algebraic expression is **simplified** when parentheses have been removed and like terms have been combined.

Study Tip

To combine like terms mentally, add or subtract the coefficients of the terms. Use this result as the coefficient of the terms' variable factor(s).

EXAMPLE 8 Simplifying an Algebraic Expression

Simplify: $6(2x^2 + 4x) + 10(4x^2 + 3x)$.

Solution

$$\begin{aligned} & 6(2x^2 + 4x) + 10(4x^2 + 3x) \\ &= 6 \cdot 2x^2 + 6 \cdot 4x + 10 \cdot 4x^2 + 10 \cdot 3x && \text{Use the distributive property to remove the parentheses.} \\ &= 12x^2 + 24x + 40x^2 + 30x && \text{Multiply.} \\ &= (12x^2 + 40x^2) + (24x + 30x) && \text{Group like terms.} \\ &= 52x^2 + 54x && \text{Combine like terms.} \end{aligned}$$

$52x^2$ and $54x$ are not like terms. They contain different variable factors, x^2 and x, and cannot be combined.

Check Point 8 Simplify: $7(4x^2 + 3x) + 2(5x^2 + x)$.

Properties of Negatives

The distributive property can be extended to cover more than two terms within parentheses. For example,

$$\begin{aligned} -3(4x - 2y + 6) &= -3 \cdot 4x - (-3) \cdot 2y - 3 \cdot 6 \\ &= -12x - (-6y) - 18 \\ &= -12x + 6y - 18. \end{aligned}$$

This sign represents subtraction. This sign tells us that the number is negative.

The voice balloons illustrate that negative signs can appear side by side. They can represent the operation of subtraction or the fact that a real number is negative. Here is a list of properties of negatives and how they are applied to algebraic expressions:

Properties of Negatives

Let a and b represent real numbers, variables, or algebraic expressions.

Property	Examples
1. $(-1)a = -a$	$(-1)4xy = -4xy$
2. $-(-a) = a$	$-(-6y) = 6y$
3. $(-a)b = -ab$	$(-7)4xy = -7 \cdot 4xy = -28xy$
4. $a(-b) = -ab$	$5x(-3y) = -5x \cdot 3y = -15xy$
5. $-(a + b) = -a - b$	$-(7x + 6y) = -7x - 6y$
6. $-(a - b) = -a + b = b - a$	$-(3x - 7y) = -3x + 7y = 7y - 3x$

It is not uncommon to see algebraic expressions with parentheses preceded by a negative sign or subtraction. Properties 5 and 6 in the box, $-(a + b) = -a - b$ and $-(a - b) = -a + b$, are related to this situation. An expression of the form $-(a + b)$ can be simplified as follows:

$$-(a + b) = -1(a + b) = (-1)a + (-1)b = -a + (-b) = -a - b.$$

Do you see a fast way to obtain the simplified expression on the right at the bottom of the previous page? **If a negative sign or a subtraction symbol appears outside parentheses, drop the parentheses and change the sign of every term within the parentheses.** For example,

$$-(3x^2 - 7x - 4) = -3x^2 + 7x + 4.$$

EXAMPLE 9 Simplifying an Algebraic Expression

Simplify: $8x + 2[5 - (x - 3)]$.

Solution

$8x + 2[5 - (x - 3)]$

$= 8x + 2[5 - x + 3]$ — Drop parentheses and change the sign of each term in parentheses: $-(x - 3) = -x + 3$.

$= 8x + 2[8 - x]$ — Simplify inside brackets: $5 + 3 = 8$.

$= 8x + 16 - 2x$ — Apply the distributive property: $2[8 - x] = 2 \cdot 8 - 2x = 16 - 2x$.

$= (8x - 2x) + 16$ — Group like terms.

$= (8 - 2)x + 16$ — Apply the distributive property.

$= 6x + 16$ — Simplify.

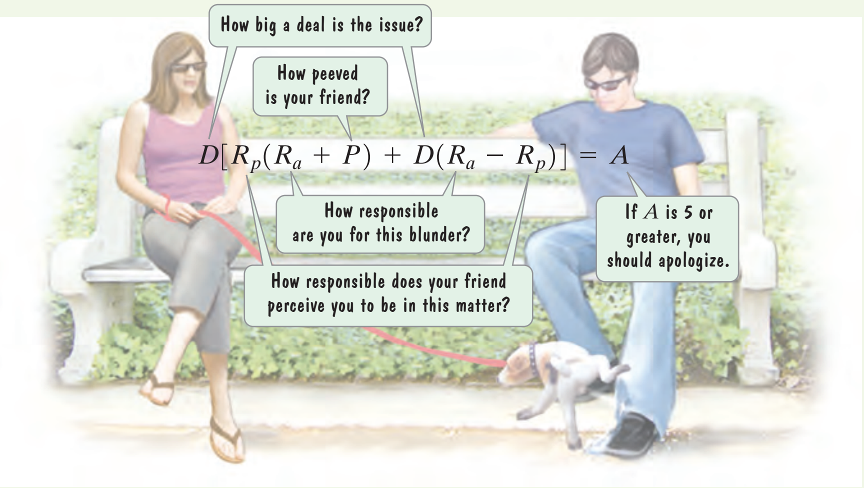
Check Point 9 Simplify: $6 + 4[7 - (x - 2)]$.

Using Algebra to Solve Problems in Your Everyday Life

Should You Apologize to Your Friend?

In this formula, each variable is assigned a number from 1 (low) to 10 (high).

In *Geek Logik* (Workman Publishing, 2006), humorist Garth Sundem presents formulas covering dating, romance, career, finance, everyday decisions, and health. On the right is a sample of one of his formulas that "takes the guesswork out of life, providing easier living through algebra."



Exercise Set P.1

Practice Exercises

In Exercises 1–16, evaluate each algebraic expression for the given value or values of the variable(s).

1. $7 + 5x$, for $x = 10$

2. $8 + 6x$, for $x = 5$

3. $6x - y$, for $x = 3$ and $y = 8$

4. $8x - y$, for $x = 3$ and $y = 4$

5. $x^2 + 3x$, for $x = 8$

6. $x^2 + 5x$, for $x = 6$

7. $x^2 - 6x + 3$, for $x = 7$

8. $x^2 - 7x + 4$, for $x = 8$

9. $4 + 5(x - 7)^3$, for $x = 9$

10. $6 + 5(x - 6)^3$, for $x = 8$

11. $x^2 - 3(x - y)$, for $x = 8$ and $y = 2$

12. $x^2 - 4(x - y)$, for $x = 8$ and $y = 3$

13. $\frac{5(x+2)}{2x-14}$, for $x = 10$

14. $\frac{7(x-3)}{2x-16}$, for $x = 9$

15. $\frac{2x+3y}{x+1}$, for $x = -2$ and $y = 4$

16. $\frac{2x+y}{xy-2x}$, for $x = -2$ and $y = 4$

The formula

$$C = \frac{5}{9}(F - 32)$$

expresses the relationship between Fahrenheit temperature, F, and Celsius temperature, C. In Exercises 17–18, use the formula to convert the given Fahrenheit temperature to its equivalent temperature on the Celsius scale.

17. 50°F

18. 86°F

A football was kicked vertically upward from a height of 4 feet with an initial speed of 60 feet per second. The formula

$$h = 4 + 60t - 16t^2$$

describes the ball's height above the ground, h, in feet, t seconds after it was kicked. Use this formula to solve Exercises 19–20.

19. What was the ball's height 2 seconds after it was kicked?

20. What was the ball's height 3 seconds after it was kicked?

In Exercises 21–28, find the intersection of the sets.

21. $\{1, 2, 3, 4\} \cap \{2, 4, 5\}$

22. $\{1, 3, 7\} \cap \{2, 3, 8\}$

23. $\{s, e, t\} \cap \{t, e, s\}$

24. $\{r, e, a, l\} \cap \{l, e, a, r\}$

25. $\{1, 3, 5, 7\} \cap \{2, 4, 6, 8, 10\}$

26. $\{0, 1, 3, 5\} \cap \{-5, -3, -1\}$

27. $\{a, b, c, d\} \cap \varnothing$

28. $\{w, y, z\} \cap \varnothing$

In Exercises 29–34, find the union of the sets.

29. $\{1, 2, 3, 4\} \cup \{2, 4, 5\}$

30. $\{1, 3, 7, 8\} \cup \{2, 3, 8\}$

31. $\{1, 3, 5, 7\} \cup \{2, 4, 6, 8, 10\}$

32. $\{0, 1, 3, 5\} \cup \{2, 4, 6\}$

33. $\{a, e, i, o, u\} \cup \varnothing$

34. $\{e, m, p, t, y\} \cup \varnothing$

In Exercises 35–38, list all numbers from the given set that are **a.** *natural numbers,* **b.** *whole numbers,* **c.** *integers,* **d.** *rational numbers,* **e.** *irrational numbers,* **f.** *real numbers.*

35. $\{-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}\}$

36. $\{-7, -0.\overline{6}, 0, \sqrt{49}, \sqrt{50}\}$

37. $\{-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}\}$

38. $\{-5, -0.\overline{3}, 0, \sqrt{2}, \sqrt{4}\}$

39. Give an example of a whole number that is not a natural number.

40. Give an example of a rational number that is not an integer.

41. Give an example of a number that is an integer, a whole number, and a natural number.

42. Give an example of a number that is a rational number, an integer, and a real number.

Determine whether each statement in Exercises 43–50 is true or false.

43. $-13 \le -2$

44. $-6 > 2$

45. $4 \ge -7$

46. $-13 < -5$

47. $-\pi \ge -\pi$

48. $-3 > -13$

49. $0 \ge -6$

50. $0 \ge -13$

In Exercises 51–60, rewrite each expression without absolute value bars.

51. $|300|$

52. $|-203|$

53. $|12 - \pi|$

54. $|7 - \pi|$

55. $|\sqrt{2} - 5|$

56. $|\sqrt{5} - 13|$

57. $\frac{-3}{|-3|}$

58. $\frac{-7}{|-7|}$

59. $||-3| - |-7||$

60. $||-5| - |-13||$

In Exercises 61–66, evaluate each algebraic expression for $x = 2$ and $y = -5$.

61. $|x + y|$

62. $|x - y|$

63. $|x| + |y|$

64. $|x| - |y|$

65. $\frac{y}{|y|}$

66. $\frac{|x|}{x} + \frac{|y|}{y}$

In Exercises 67–74, express the distance between the given numbers using absolute value. Then find the distance by evaluating the absolute value expression.

67. 2 and 17

68. 4 and 15

69. −2 and 5

70. −6 and 8

71. −19 and −4

72. −26 and −3

73. −3.6 and −1.4

74. −5.4 and −1.2

In Exercises 75–84, state the name of the property illustrated.

75. $6 + (-4) = (-4) + 6$

76. $11 \cdot (7 + 4) = 11 \cdot 7 + 11 \cdot 4$

77. $6 + (2 + 7) = (6 + 2) + 7$

78. $6 \cdot (2 \cdot 3) = 6 \cdot (3 \cdot 2)$

79. $(2 + 3) + (4 + 5) = (4 + 5) + (2 + 3)$

80. $7 \cdot (11 \cdot 8) = (11 \cdot 8) \cdot 7$

81. $2(-8 + 6) = -16 + 12$

82. $-8(3 + 11) = -24 + (-88)$

83. $\frac{1}{(x+3)}(x + 3) = 1, x \ne -3$

84. $(x + 4) + [-(x + 4)] = 0$

In Exercises 85–96, simplify each algebraic expression.

85. $5(3x + 4) - 4$

86. $2(5x + 4) - 3$

87. $5(3x - 2) + 12x$

88. $2(5x - 1) + 14x$

89. $7(3y - 5) + 2(4y + 3)$

90. $4(2y - 6) + 3(5y + 10)$

91. $5(3y - 2) - (7y + 2)$

92. $4(5y - 3) - (6y + 3)$

93. $7 - 4[3 - (4y - 5)]$

94. $6 - 5[8 - (2y - 4)]$

95. $18x^2 + 4 - [6(x^2 - 2) + 5]$

96. $14x^2 + 5 - [7(x^2 - 2) + 4]$

In Exercises 97–102, write each algebraic expression without parentheses.

97. $-(-14x)$

98. $-(-17y)$

99. $-(2x - 3y - 6)$

100. $-(5x - 13y - 1)$

101. $\frac{1}{3}(3x) + [(4y) + (-4y)]$

102. $\frac{1}{2}(2y) + [(-7x) + 7x]$

Practice Plus

In Exercises 103–110, insert either <, >, or = in the shaded area to make a true statement.

103. $|-6| \;\square\; |-3|$

104. $|-20| \;\square\; |-50|$

105. $\left|\frac{3}{5}\right| \;\square\; |-0.6|$

106. $\left|\frac{5}{2}\right| \;\square\; |-2.5|$

107. $\frac{30}{40} - \frac{3}{4} \;\square\; \frac{14}{15} \cdot \frac{15}{14}$

108. $\frac{17}{18} \cdot \frac{18}{17} \;\square\; \frac{50}{60} - \frac{5}{6}$

109. $\frac{8}{13} \div \frac{8}{13} \;\square\; |-1|$

110. $|-2| \;\square\; \frac{4}{17} \div \frac{4}{17}$

In Exercises 111–120, use the order of operations to simplify each expression.

111. $8^2 - 16 \div 2^2 \cdot 4 - 3$

112. $10^2 - 100 \div 5^2 \cdot 2 - 3$

113. $\dfrac{5 \cdot 2 - 3^2}{[3^2 - (-2)]^2}$

114. $\dfrac{10 \div 2 + 3 \cdot 4}{(12 - 3 \cdot 2)^2}$

115. $8 - 3[-2(2 - 5) - 4(8 - 6)]$

116. $8 - 3[-2(5 - 7) - 5(4 - 2)]$

117. $\dfrac{2(-2) - 4(-3)}{5 - 8}$

118. $\dfrac{6(-4) - 5(-3)}{9 - 10}$

119. $\dfrac{(5 - 6)^2 - 2|3 - 7|}{89 - 3 \cdot 5^2}$

120. $\dfrac{12 \div 3 \cdot 5|2^2 + 3^2|}{7 + 3 - 6^2}$

In Exercises 121–128, write each English phrase as an algebraic expression. Then simplify the expression. Let x represent the number.

121. A number decreased by the sum of the number and four

122. A number decreased by the difference between eight and the number

123. Six times the product of negative five and a number

124. Ten times the product of negative four and a number

125. The difference between the product of five and a number and twice the number

126. The difference between the product of six and a number and negative two times the number

127. The difference between eight times a number and six more than three times the number

128. Eight decreased by three times the sum of a number and six

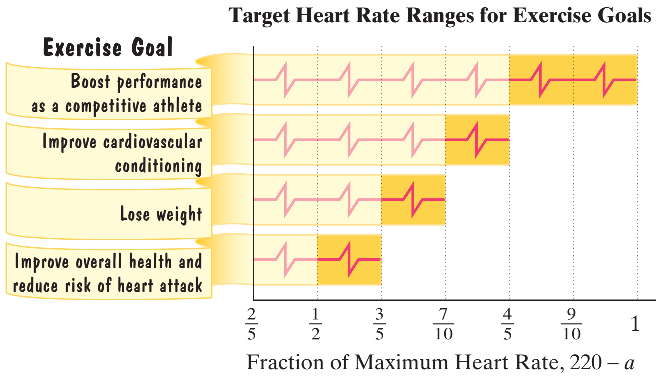
Application Exercises

The maximum heart rate, in beats per minute, that you should achieve during exercise is 220 minus your age:

$$220 - a.$$

This algebraic expression gives maximum heart rate in terms of age, a.

The bar graph at the top of the next column shows the target heart rate ranges for four types of exercise goals. The lower and upper limits of these ranges are fractions of the maximum heart rate, 220 − a. Exercises 129–130 are based on the information in the graph.



129. If your exercise goal is to improve cardiovascular conditioning, the graph shows the following range for target heart rate, H, in beats per minute:

Lower limit of range: $H = \frac{7}{10}(220 - a)$

Upper limit of range: $H = \frac{4}{5}(220 - a)$.

a. What is the lower limit of the heart range, in beats per minute, for a 20-year-old with this exercise goal?

b. What is the upper limit of the heart range, in beats per minute, for a 20-year-old with this exercise goal?

130. If your exercise goal is to improve overall health, the graph shows the following range for target heart rate, H, in beats per minute:

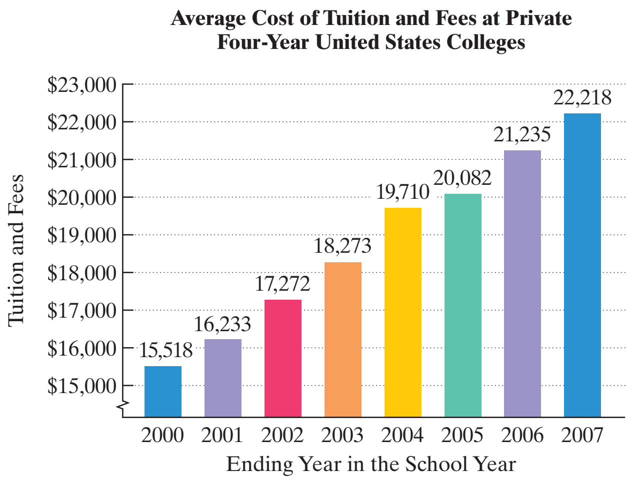
Lower limit of range: $H = \frac{1}{2}(220 - a)$

Upper limit of range: $H = \frac{3}{5}(220 - a)$.

a. What is the lower limit of the heart range, in beats per minute, for a 30-year-old with this exercise goal?

b. What is the upper limit of the heart range, in beats per minute, for a 30-year-old with this exercise goal?

The bar graph shows the average cost of tuition and fees at private four-year colleges in the United States.



Source: The College Board

The formula

$$T = 15{,}395 + 988x - 2x^2$$

models the average cost of tuition and fees, T, at private U.S. colleges for the school year ending x years after 2000. Use this information to solve Exercises 131–132.

131. **a.** Use the formula to find the average cost of tuition and fees at private U.S. colleges for the school year ending in 2007.

b. By how much does the formula underestimate or overestimate the actual cost shown by the graph at the bottom of the previous page for the school year ending in 2007?

c. Use the formula to project the average cost of tuition and fees at private U.S. colleges for the school year ending in 2010.

132. **a.** Use the formula to find the average cost of tuition and fees at private U.S. colleges for the school year ending in 2006.

b. By how much does the formula underestimate or overestimate the actual cost shown by the graph at the bottom of the previous page for the school year ending in 2006?

c. Use the formula to project the average cost of tuition and fees at private U.S. colleges for the school year ending in 2012.

133. You had \$10,000 to invest. You put x dollars in a safe, government-insured certificate of deposit paying 5% per year. You invested the remainder of the money in noninsured corporate bonds paying 12% per year. Your total interest earned at the end of the year is given by the algebraic expression

$$0.05x + 0.12(10{,}000 - x).$$

a. Simplify the algebraic expression.

b. Use each form of the algebraic expression to determine your total interest earned at the end of the year if you invested \$6000 in the safe, government-insured certificate of deposit.

134. It takes you 50 minutes to get to campus. You spend t minutes walking to the bus stop and the rest of the time riding the bus. Your walking rate is 0.06 mile per minute and the bus travels at a rate of 0.5 mile per minute. The total distance walking and traveling by bus is given by the algebraic expression

$$0.06t + 0.5(50 - t).$$

a. Simplify the algebraic expression.

b. Use each form of the algebraic expression to determine the total distance that you travel if you spend 20 minutes walking to the bus stop.

Writing in Mathematics

Writing about mathematics will help you learn mathematics. For all writing exercises in this book, use complete sentences to respond to the question. Some writing exercises can be answered in a sentence; others require a paragraph or two. You can decide how much you need to write as long as your writing clearly and directly answers the question in the exercise. Standard references such as a dictionary and a thesaurus should be helpful.

135. What is an algebraic expression? Give an example with your explanation.

136. If n is a natural number, what does b^n mean? Give an example with your explanation.

137. What does it mean when we say that a formula models real-world phenomena?

138. What is the intersection of sets A and B?

139. What is the union of sets A and B?

140. How do the whole numbers differ from the natural numbers?

141. Can a real number be both rational and irrational? Explain your answer.

142. If you are given two real numbers, explain how to determine which is the lesser.

143. Think of a situation where you either apologized or did not apologize for a blunder you committed. Use the formula in the essay on page 15 to determine whether or not you should have apologized. How accurately does the formula model what you actually did?

144. Read *Geek Logik* by Garth Sundem (Workman Publishing, 2006). Would you recommend the book to college algebra students? Were you amused by the humor? Which formulas did you find most useful? Apply at least one of the formulas by plugging your life data into the equation, using the order of operations, and coming up with an answer to one of life's dilemmas.

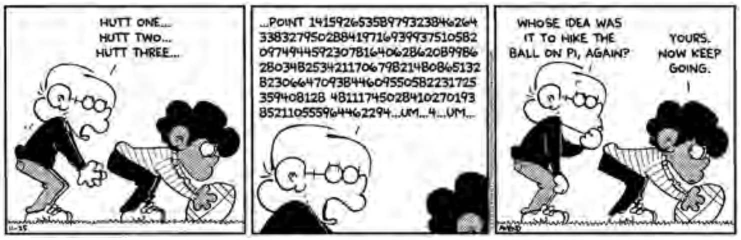
Critical Thinking Exercises

Make Sense? *In Exercises 145–148, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

145. My mathematical model describes the data for tuition and fees at public four-year colleges for the past ten years extremely well, so it will serve as an accurate prediction for the cost of public colleges in 2050.

146. A model that describes the average cost of tuition and fees at private U.S. colleges for the school year ending x years after 2000 cannot be used to estimate the cost of private education for the school year ending in 2000.

147. The humor in this cartoon is based on the fact that the football will never be hiked.



148. Just as the commutative properties change groupings, the associative properties change order.

In Exercises 149–156, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

149. Every rational number is an integer.

150. Some whole numbers are not integers.

151. Some rational numbers are not positive.

152. Irrational numbers cannot be negative.

153. The term x has no coefficient.

154. $5 + 3(x - 4) = 8(x - 4) = 8x - 32$

155. $-x - x = -x + (-x) = 0$

156. $x - 0.02(x + 200) = 0.98x - 4$

In Exercises 157–159, insert either < or > in the shaded area between the numbers to make the statement true.

157. $\sqrt{2}$ ▭ 1.5

158. $-\pi$ ▭ -3.5

159. $-\frac{3.14}{2}$ ▭ $-\frac{\pi}{2}$

Preview Exercises

Exercises 160–162 will help you prepare for the material covered in the next section.

160. In parts (a) and (b), complete each statement.

a. $b^4 \cdot b^3 = (b \cdot b \cdot b \cdot b)(b \cdot b \cdot b) = b^?$

b. $b^5 \cdot b^5 = (b \cdot b \cdot b \cdot b \cdot b)(b \cdot b \cdot b \cdot b \cdot b) = b^?$

c. Generalizing from parts (a) and (b), what should be done with the exponents when multiplying exponential expressions with the same base?

161. In parts (a) and (b), complete each statement.

a. $\frac{b^7}{b^3} = \frac{b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b \cdot b} = b^?$

b. $\frac{b^8}{b^2} = \frac{b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b} = b^?$

c. Generalizing from parts (a) and (b), what should be done with the exponents when dividing exponential expressions with the same base?

162. If 6.2 is multiplied by 10^3, what does this multiplication do to the decimal point in 6.2?

Section P.2 Exponents and Scientific Notation

Objectives

1. Use properties of exponents.
2. Simplify exponential expressions.
3. Use scientific notation.

Listening to the radio on the way to campus, you hear politicians discussing the problem of the national debt, which exceeds $9 trillion. They state that it's more than the gross domestic product of China, the world's second-richest nation, and four times greater than the combined net worth of America's 691 billionaires. They make it seem like the national debt is a real problem, but later you realize that you don't really know what a number like 9 trillion means. If the national debt were evenly divided among all citizens of the country, how much would every man, woman, and child have to pay? Is economic doomsday about to arrive?

In this section, you will learn to use exponents to provide a way of putting large and small numbers in perspective. Using this skill, we will explore the meaning of the national debt.

1 Use properties of exponents.

Properties of Exponents

The major properties of exponents are summarized in the box that follows and continues on the next page.

Properties of Exponents

Property	Examples
The Negative-Exponent Rule If b is any real number other than 0 and n is a natural number, then $b^{-n} = \frac{1}{b^n}$.	• $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$ • $\frac{1}{4^{-2}} = \frac{1}{\frac{1}{4^2}} = 4^2 = 16$

Study Tip

When a negative integer appears as an exponent, switch the position of the base (from numerator to denominator or from denominator to numerator) and make the exponent positive.