Section P.4

Objectives

1. Understand the vocabulary of polynomials.
2. Add and subtract polynomials.
3. Multiply polynomials.
4. Use FOIL in polynomial multiplication.
5. Use special products in polynomial multiplication.
6. Perform operations with polynomials in several variables.

Polynomials

Can that be Axl, your author's yellow lab, sharing a special moment with a baby chick? And if it is (it is), what possible relevance can this have to polynomials? An answer is promised before you reach the exercise set. For now, we open the section by defining and describing polynomials.

Old Dog . . . New Chicks

How We Define Polynomials

More education results in a higher income. The mathematical models

$$M = -18x^3 + 923x^2 - 9603x + 48{,}446$$

$$\text{and} \quad W = 17x^3 - 450x^2 + 6392x - 14{,}764$$

describe the median, or middlemost, annual income for men, M, and women, W, who have completed x years of education. We'll be working with these models and the data upon which they are based in the exercise set.

The algebraic expressions that appear on the right sides of the models are examples of *polynomials*. A **polynomial** is a single term or the sum of two or more terms containing variables with whole-number exponents. The polynomials above each contain four terms. Equations containing polynomials are used in such diverse areas as science, business, medicine, psychology, and sociology. In this section, we review basic ideas about polynomials and their operations.

1 Understand the vocabulary of polynomials.

How We Describe Polynomials

Consider the polynomial

$$7x^3 - 9x^2 + 13x - 6.$$

We can express this polynomial as

$$7x^3 + (-9x^2) + 13x + (-6).$$

The polynomial contains four terms. It is customary to write the terms in the order of descending powers of the variable. This is the **standard form** of a polynomial.

Some polynomials contain only one variable. Each term of such a polynomial in x is of the form ax^n. If $a \neq 0$, the **degree** of ax^n is n. For example, the degree of the term $7x^3$ is 3.

Study Tip

We can express 0 in many ways, including $0x, 0x^2$, and $0x^3$. It is impossible to assign a unique exponent to the variable. This is why 0 has no defined degree.

The Degree of ax^n

If $a \neq 0$, the degree of ax^n is n. The degree of a nonzero constant is 0. The constant 0 has no defined degree.

Here is an example of a polynomial and the degree of each of its four terms:

$$6x^4 - 3x^3 + 2x - 5.$$

degree 4 | degree 3 | degree 1 | degree of nonzero constant: 0

Notice that the exponent on x for the term $2x$, meaning $2x^1$, is understood to be 1. For this reason, the degree of $2x$ is 1. You can think of -5 as $-5x^0$; thus, its degree is 0.

A polynomial is simplified when it contains no grouping symbols and no like terms. A simplified polynomial that has exactly one term is called a **monomial**. A **binomial** is a simplified polynomial that has two terms. A **trinomial** is a simplified polynomial with three terms. Simplified polynomials with four or more terms have no special names.

The **degree of a polynomial** is the greatest degree of all the terms of the polynomial. For example, $4x^2 + 3x$ is a binomial of degree 2 because the degree of the first term is 2, and the degree of the other term is less than 2. Also, $7x^5 - 2x^2 + 4$ is a trinomial of degree 5 because the degree of the first term is 5, and the degrees of the other terms are less than 5.

Up to now, we have used x to represent the variable in a polynomial. However, any letter can be used. For example,

- $7x^5 - 3x^3 + 8$ is a polynomial (in x) of degree 5. Because there are three terms, the polynomial is a trinomial.
- $6y^3 + 4y^2 - y + 3$ is a polynomial (in y) of degree 3. Because there are four terms, the polynomial has no special name.
- $z^7 + \sqrt{2}$ is a polynomial (in z) of degree 7. Because there are two terms, the polynomial is a binomial.

We can tie together the threads of our discussion with the formal definition of a polynomial in one variable. In this definition, the coefficients of the terms are represented by a_n (read "a sub n"), a_{n-1} (read "a sub n minus 1"), a_{n-2}, and so on. The small letters to the lower right of each a are called **subscripts** and are *not exponents*. Subscripts are used to distinguish one constant from another when a large and undetermined number of such constants are needed.

Definition of a Polynomial in *x*

A **polynomial in *x*** is an algebraic expression of the form

$$a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1x + a_0,$$

where $a_n, a_{n-1}, a_{n-2}, \ldots, a_1$, and a_0 are real numbers, $a_n \neq 0$, and n is a nonnegative integer. The polynomial is of **degree *n*,** a_n is the **leading coefficient**, and a_0 is the **constant term**.

2 Add and subtract polynomials.

Adding and Subtracting Polynomials

Polynomials are added and subtracted by combining like terms. For example, we can combine the monomials $-9x^3$ and $13x^3$ using addition as follows:

$$-9x^3 + 13x^3 = (-9 + 13)x^3 = 4x^3.$$

These like terms both contain x to the third power.

Add coefficients and keep the same variable factor, x^3.

EXAMPLE 1 Adding and Subtracting Polynomials

Perform the indicated operations and simplify:

a. $(-9x^3 + 7x^2 - 5x + 3) + (13x^3 + 2x^2 - 8x - 6)$

b. $(7x^3 - 8x^2 + 9x - 6) - (2x^3 - 6x^2 - 3x + 9)$.

Solution

a. $(-9x^3 + 7x^2 - 5x + 3) + (13x^3 + 2x^2 - 8x - 6)$

$= (-9x^3 + 13x^3) + (7x^2 + 2x^2) + (-5x - 8x) + (3 - 6)$ Group like terms.

$= 4x^3 + 9x^2 + (-13x) + (-3)$ Combine like terms.

$= 4x^3 + 9x^2 - 13x - 3$ Simplify.

b. $(7x^3 - 8x^2 + 9x - 6) - (2x^3 - 6x^2 - 3x + 9)$

Change the sign of each coefficient.

$= (7x^3 - 8x^2 + 9x - 6) + (-2x^3 + 6x^2 + 3x - 9)$ Rewrite subtraction as addition of the additive inverse.

$= (7x^3 - 2x^3) + (-8x^2 + 6x^2) + (9x + 3x) + (-6 - 9)$ Group like terms.

$= 5x^3 + (-2x^2) + 12x + (-15)$ Combine like terms.

$= 5x^3 - 2x^2 + 12x - 15$ Simplify.

Study Tip

You can also arrange like terms in columns and combine vertically:

$$\begin{array}{r} 7x^3 - 8x^2 + 9x - 6 \\ -2x^3 + 6x^2 + 3x - 9 \\ \hline 5x^3 - 2x^2 + 12x - 15 \end{array}$$

The like terms can be combined by adding their coefficients and keeping the same variable factor.

Check Point 1 Perform the indicated operations and simplify:

a. $(-17x^3 + 4x^2 - 11x - 5) + (16x^3 - 3x^2 + 3x - 15)$

b. $(13x^3 - 9x^2 - 7x + 1) - (-7x^3 + 2x^2 - 5x + 9)$.

3 Multiply polynomials.

Multiplying Polynomials

The product of two monomials is obtained by using properties of exponents. For example,

$$(-8x^6)(5x^3) = -8 \cdot 5x^{6+3} = -40x^9.$$

Multiply coefficients and add exponents.

Furthermore, we can use the distributive property to multiply a monomial and a polynomial that is not a monomial. For example,

$$\underbrace{3x^4}_{\text{Monomial}}\underbrace{(2x^3 - 7x + 3)}_{\text{Trinomial}} = 3x^4 \cdot 2x^3 - 3x^4 \cdot 7x + 3x^4 \cdot 3 = 6x^7 - 21x^5 + 9x^4.$$

How do we multiply two polynomials if neither is a monomial? For example, consider

$$\underbrace{(2x + 3)}_{\text{Binomial}}\underbrace{(x^2 + 4x + 5)}_{\text{Trinomial}}.$$

Study Tip

Don't confuse adding and multiplying monomials.

Addition:

$$5x^4 + 6x^4 = 11x^4$$

Multiplication:

$$\begin{aligned}(5x^4)(6x^4) &= (5 \cdot 6)(x^4 \cdot x^4) \\ &= 30x^{4+4} \\ &= 30x^8\end{aligned}$$

Only like terms can be added or subtracted, but unlike terms may be multiplied.

Addition:

$5x^4 + 3x^2$ cannot be simplified.

Multiplication:

$$\begin{aligned}(5x^4)(3x^2) &= (5 \cdot 3)(x^4 \cdot x^2) \\ &= 15x^{4+2} \\ &= 15x^6\end{aligned}$$

One way to perform $(2x + 3)(x^2 + 4x + 5)$ is to distribute $2x$ throughout the trinomial

$$2x(x^2 + 4x + 5)$$

and 3 throughout the trinomial

$$3(x^2 + 4x + 5).$$

Then combine the like terms that result.

Multiplying Polynomials When Neither Is a Monomial

Multiply each term of one polynomial by each term of the other polynomial. Then combine like terms.

EXAMPLE 2 Multiplying a Binomial and a Trinomial

Multiply: $(2x + 3)(x^2 + 4x + 5)$.

Solution

$$(2x + 3)(x^2 + 4x + 5)$$

$$= 2x(x^2 + 4x + 5) + 3(x^2 + 4x + 5)$$ Multiply the trinomial by each term of the binomial.

$$= 2x \cdot x^2 + 2x \cdot 4x + 2x \cdot 5 + 3x^2 + 3 \cdot 4x + 3 \cdot 5$$ Use the distributive property.

$$= 2x^3 + 8x^2 + 10x + 3x^2 + 12x + 15$$ Multiply monomials: Multiply coefficients and add exponents.

$$= 2x^3 + 11x^2 + 22x + 15$$ Combine like terms: $8x^2 + 3x^2 = 11x^2$ and $10x + 12x = 22x$.

Another method for performing the multiplication is to use a vertical format similar to that used for multiplying whole numbers.

$$\begin{array}{rrrr}
 & x^2 & + 4x & + 5 \\
 & & 2x & + 3 \\
\hline
 & 3x^2 & + 12x & + 15 \\
2x^3 + & 8x^2 & + 10x & \\
\hline
2x^3 + & 11x^2 & + 22x & + 15
\end{array}$$

$3(x^2 + 4x + 5)$

$2x(x^2 + 4x + 5)$

Write like terms in the same column.

Combine like terms.

Check Point 2 Multiply: $(5x - 2)(3x^2 - 5x + 4)$.

4 Use FOIL in polynomial multiplication.

The Product of Two Binomials: FOIL

Frequently, we need to find the product of two binomials. One way to perform this multiplication is to distribute each term in the first binomial through the second binomial. For example, we can find the product of the binomials $3x + 2$ and $4x + 5$ as follows:

$$(3x + 2)(4x + 5) = 3x(4x + 5) + 2(4x + 5)$$
$$= 3x(4x) + 3x(5) + 2(4x) + 2(5)$$
$$= 12x^2 + 15x + 8x + 10.$$

Distribute $3x$ over $4x + 5$.

Distribute 2 over $4x + 5$.

We'll combine these like terms later. For now, our interest is in how to obtain *each* of these four terms.

We can also find the product of $3x + 2$ and $4x + 5$ using a method called FOIL, which is based on our work shown at the bottom of the previous page. Any two binomials can be quickly multiplied by using the FOIL method, in which **F** represents the product of the **first** terms in each binomial, **O** represents the product of the **outside** terms, **I** represents the product of the **inside** terms, and **L** represents the product of the **last**, or second, terms in each binomial. For example, we can use the FOIL method to find the product of the binomials $3x + 2$ and $4x + 5$ as follows:

first last

$$(3x + 2)(4x + 5) = \overset{\text{F}}{\underbrace{12x^2}_{\text{Product of First terms}}} + \overset{\text{O}}{\underbrace{15x}_{\text{Product of Outside terms}}} + \overset{\text{I}}{\underbrace{8x}_{\text{Product of Inside terms}}} + \overset{\text{L}}{\underbrace{10}_{\text{Product of Last terms}}}$$

inside outside

$$= 12x^2 + 23x + 10 \qquad \text{Combine like terms.}$$

In general, here's how to use the FOIL method to find the product of $ax + b$ and $cx + d$:

Using the FOIL Method to Multiply Binomials

first last

$$(ax + b)(cx + d) = \overset{\text{F}}{\underbrace{ax \cdot cx}_{\text{Product of First terms}}} + \overset{\text{O}}{\underbrace{ax \cdot d}_{\text{Product of Outside terms}}} + \overset{\text{I}}{\underbrace{b \cdot cx}_{\text{Product of Inside terms}}} + \overset{\text{L}}{\underbrace{b \cdot d}_{\text{Product of Last terms}}}$$

inside outside

EXAMPLE 3 Using the FOIL Method

Multiply: $(3x + 4)(5x - 3)$.

Solution

first last

$$(3x + 4)(5x - 3) = \overset{\text{F}}{3x \cdot 5x} + \overset{\text{O}}{3x(-3)} + \overset{\text{I}}{4 \cdot 5x} + \overset{\text{L}}{4(-3)}$$

inside outside

$$= 15x^2 - 9x + 20x - 12$$

$$= 15x^2 + 11x - 12 \qquad \text{Combine like terms.}$$

Check Point 3 Multiply: $(7x - 5)(4x - 3)$.

5 Use special products in polynomial multiplication.

Special Products

There are several products that occur so frequently that it's convenient to memorize the form, or pattern, of these formulas.

Special Products

Let A and B represent real numbers, variables, or algebraic expressions.

Special Product	**Example**
Sum and Difference of Two Terms	
$(A + B)(A - B) = A^2 - B^2$	$(2x + 3)(2x - 3) = (2x)^2 - 3^2$ $= 4x^2 - 9$
Squaring a Binomial	
$(A + B)^2 = A^2 + 2AB + B^2$	$(y + 5)^2 = y^2 + 2 \cdot y \cdot 5 + 5^2$ $= y^2 + 10y + 25$
$(A - B)^2 = A^2 - 2AB + B^2$	$(3x - 4)^2$ $= (3x)^2 - 2 \cdot 3x \cdot 4 + 4^2$ $= 9x^2 - 24x + 16$
Cubing a Binomial	
$(A + B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$	$(x + 4)^3$ $= x^3 + 3x^2(4) + 3x(4)^2 + 4^3$ $= x^3 + 12x^2 + 48x + 64$
$(A - B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$	$(x - 2)^3$ $= x^3 - 3x^2(2) + 3x(2)^2 - 2^3$ $= x^3 - 6x^2 + 12x - 8$

Study Tip

Although it's convenient to memorize these forms, the FOIL method can be used on all five examples in the box. To cube $x + 4$, you can first square $x + 4$ using FOIL and then multiply this result by $x + 4$. In short, you do not necessarily have to utilize these special formulas. What is the advantage of knowing and using these forms?

6 Perform operations with polynomials in several variables.

Polynomials in Several Variables

A **polynomial in two variables**, x and y, contains the sum of one or more monomials in the form ax^ny^m. The constant, a, is the **coefficient**. The exponents, n and m, represent whole numbers. The **degree** of the monomial ax^ny^m is $n + m$.

Here is an example of a polynomial in two variables:

The coefficients are 7, −17, 1, −6, and 9.

$$7x^2y^3 - 17x^4y^2 + xy - 6y^2 + 9.$$

Degree of monomial: $2 + 3 = 5$

Degree of monomial: $4 + 2 = 6$

Degree of monomial $(1x^1y^1)$: $1 + 1 = 2$

Degree of monomial $(-6x^0y^2)$: $0 + 2 = 2$

Degree of monomial $(9x^0y^0)$: $0 + 0 = 0$

The **degree of a polynomial in two variables** is the highest degree of all its terms. For the preceding polynomial, the degree is 6.

Polynomials containing two or more variables can be added, subtracted, and multiplied just like polynomials that contain only one variable. For example, we can add the monomials $-7xy^2$ and $13xy^2$ as follows:

$$-7xy^2 + 13xy^2 = (-7 + 13)xy^2 = 6xy^2.$$

These like terms both contain the variable factors x and y^2.

Add coefficients and keep the same variable factors, xy^2.

EXAMPLE 4 Multiplying Polynomials in Two Variables

Multiply: **a.** $(x + 4y)(3x - 5y)$ **b.** $(5x + 3y)^2$.

Solution We will perform the multiplication in part (a) using the FOIL method. We will multiply in part (b) using the formula for the square of a binomial sum, $(A + B)^2$.

a. $(x + 4y)(3x - 5y)$ Multiply these binomials using the FOIL method.

F O I L

$$= (x)(3x) + (x)(-5y) + (4y)(3x) + (4y)(-5y)$$
$$= 3x^2 - 5xy + 12xy - 20y^2$$
$$= 3x^2 + 7xy - 20y^2 \quad \text{Combine like terms.}$$

$$(A + B)^2 = A^2 + 2 \cdot A \cdot B + B^2$$

b. $(5x + 3y)^2 = (5x)^2 + 2(5x)(3y) + (3y)^2$
$$= 25x^2 + 30xy + 9y^2$$

Check Point 4 Multiply:

a. $(7x - 6y)(3x - y)$ **b.** $(2x + 4y)^2$.

Special products can sometimes be used to find the products of certain trinomials, as illustrated in Example 5.

EXAMPLE 5 Using the Special Products

Multiply: **a.** $(7x + 5 + 4y)(7x + 5 - 4y)$ **b.** $(3x + y + 1)^2$.

Solution

a. By grouping the first two terms within each of the parentheses, we can find the product using the form for the sum and difference of two terms.

$$(A + B) \cdot (A - B) = A^2 - B^2$$

$$[(7x + 5) + 4y] \cdot [(7x + 5) - 4y] = (7x + 5)^2 - (4y)^2$$
$$= (7x)^2 + 2 \cdot 7x \cdot 5 + 5^2 - (4y)^2$$
$$= 49x^2 + 70x + 25 - 16y^2$$

b. We can group the terms of $(3x + y + 1)^2$ so that the formula for the square of a binomial can be applied.

$$(A + B)^2 = A^2 + 2 \cdot A \cdot B + B^2$$

$$[(3x + y) + 1]^2 = (3x + y)^2 + 2 \cdot (3x + y) \cdot 1 + 1^2$$
$$= 9x^2 + 6xy + y^2 + 6x + 2y + 1$$

Check Point 5 Multiply:

a. $(3x + 2 + 5y)(3x + 2 - 5y)$ **b.** $(2x + y + 3)^2$.

Labrador Retrievers and Polynomial Multiplication

The color of a Labrador retriever is determined by its pair of genes. A single gene is inherited at random from each parent. The black-fur gene, B, is dominant. The yellow-fur gene, Y, is recessive. This means that labs with at least one black-fur gene (BB or BY) have black coats. Only labs with two yellow-fur genes (YY) have yellow coats.

Axl, your author's yellow lab, inherited his genetic makeup from two black BY parents.

Second BY parent, a black lab with a recessive yellow-fur gene

First BY parent, a black lab with a recessive yellow-fur gene

	B	Y
B	BB	BY
Y	BY	YY

The table shows the four possible combinations of color genes that BY parents can pass to their offspring.

Because YY is one of four possible outcomes, the probability that a yellow lab like Axl will be the offspring of these black parents is $\frac{1}{4}$.

The probabilities suggested by the table can be modeled by the expression $\left(\frac{1}{2}B + \frac{1}{2}Y\right)^2$.

$$\left(\frac{1}{2}B + \frac{1}{2}Y\right)^2 = \left(\frac{1}{2}B\right)^2 + 2\left(\frac{1}{2}B\right)\left(\frac{1}{2}Y\right) + \left(\frac{1}{2}Y\right)^2$$

$$= \frac{1}{4}BB + \frac{1}{2}BY + \frac{1}{4}YY$$

The probability of a black lab with two dominant black genes is $\frac{1}{4}$.

The probability of a black lab with a recessive yellow gene is $\frac{1}{2}$.

The probability of a yellow lab with two recessive yellow genes is $\frac{1}{4}$.

Exercise Set P.4

Practice Exercises

In Exercises 1–4, is the algebraic expression a polynomial? If it is, write the polynomial in standard form.

1. $2x + 3x^2 - 5$

2. $2x + 3x^{-1} - 5$

3. $\dfrac{2x + 3}{x}$

4. $x^2 - x^3 + x^4 - 5$

In Exercises 5–8, find the degree of the polynomial.

5. $3x^2 - 5x + 4$

6. $-4x^3 + 7x^2 - 11$

7. $x^2 - 4x^3 + 9x - 12x^4 + 63$

8. $x^2 - 8x^3 + 15x^4 + 91$

In Exercises 9–14, perform the indicated operations. Write the resulting polynomial in standard form and indicate its degree.

9. $(-6x^3 + 5x^2 - 8x + 9) + (17x^3 + 2x^2 - 4x - 13)$

10. $(-7x^3 + 6x^2 - 11x + 13) + (19x^3 - 11x^2 + 7x - 17)$

11. $(17x^3 - 5x^2 + 4x - 3) - (5x^3 - 9x^2 - 8x + 11)$

12. $(18x^4 - 2x^3 - 7x + 8) - (9x^4 - 6x^3 - 5x + 7)$

13. $(5x^2 - 7x - 8) + (2x^2 - 3x + 7) - (x^2 - 4x - 3)$

14. $(8x^2 + 7x - 5) - (3x^2 - 4x) - (-6x^3 - 5x^2 + 3)$

In Exercises 15–82, find each product.

15. $(x + 1)(x^2 - x + 1)$

16. $(x + 5)(x^2 - 5x + 25)$

17. $(2x - 3)(x^2 - 3x + 5)$

18. $(2x - 1)(x^2 - 4x + 3)$

19. $(x + 7)(x + 3)$

20. $(x + 8)(x + 5)$

21. $(x - 5)(x + 3)$

22. $(x - 1)(x + 2)$

23. $(3x + 5)(2x + 1)$

24. $(7x + 4)(3x + 1)$

25. $(2x - 3)(5x + 3)$

26. $(2x - 5)(7x + 2)$

27. $(5x^2 - 4)(3x^2 - 7)$

28. $(7x^2 - 2)(3x^2 - 5)$

29. $(8x^3 + 3)(x^2 - 5)$

30. $(7x^3 + 5)(x^2 - 2)$

31. $(x + 3)(x - 3)$

32. $(x + 5)(x - 5)$

33. $(3x + 2)(3x - 2)$

34. $(2x + 5)(2x - 5)$

35. $(5 - 7x)(5 + 7x)$

36. $(4 - 3x)(4 + 3x)$

37. $(4x^2 + 5x)(4x^2 - 5x)$

38. $(3x^2 + 4x)(3x^2 - 4x)$

39. $(1 - y^5)(1 + y^5)$

40. $(2 - y^5)(2 + y^5)$

41. $(x + 2)^2$

42. $(x + 5)^2$

43. $(2x + 3)^2$

44. $(3x + 2)^2$

45. $(x - 3)^2$

46. $(x - 4)^2$

47. $(4x^2 - 1)^2$

48. $(5x^2 - 3)^2$

49. $(7 - 2x)^2$

50. $(9 - 5x)^2$

51. $(x + 1)^3$

52. $(x + 2)^3$

53. $(2x + 3)^3$

54. $(3x + 4)^3$

55. $(x - 3)^3$

56. $(x - 1)^3$

57. $(3x - 4)^3$

58. $(2x - 3)^3$

59. $(x + 5y)(7x + 3y)$

60. $(x + 9y)(6x + 7y)$

61. $(x - 3y)(2x + 7y)$

62. $(3x - y)(2x + 5y)$

63. $(3xy - 1)(5xy + 2)$

64. $(7x^2y + 1)(2x^2y - 3)$

65. $(7x + 5y)^2$

66. $(9x + 7y)^2$

67. $(x^2y^2 - 3)^2$

68. $(x^2y^2 - 5)^2$

69. $(x - y)(x^2 + xy + y^2)$

70. $(x + y)(x^2 - xy + y^2)$

71. $(3x + 5y)(3x - 5y)$

72. $(7x + 3y)(7x - 3y)$

73. $(x + y + 3)(x + y - 3)$

74. $(x + y + 5)(x + y - 5)$

75. $(3x + 7 - 5y)(3x + 7 + 5y)$

76. $(5x + 7y - 2)(5x + 7y + 2)$

77. $[5y - (2x + 3)][5y + (2x + 3)]$

78. $[8y + (7 - 3x)][8y - (7 - 3x)]$

79. $(x + y + 1)^2$

80. $(x + y + 2)^2$

81. $(2x + y + 1)^2$

82. $(5x + 1 + 6y)^2$

Practice Plus

In Exercises 83–90, perform the indicated operation or operations.

83. $(3x + 4y)^2 - (3x - 4y)^2$

84. $(5x + 2y)^2 - (5x - 2y)^2$

85. $(5x - 7)(3x - 2) - (4x - 5)(6x - 1)$

86. $(3x + 5)(2x - 9) - (7x - 2)(x - 1)$

87. $(2x + 5)(2x - 5)(4x^2 + 25)$

88. $(3x + 4)(3x - 4)(9x^2 + 16)$

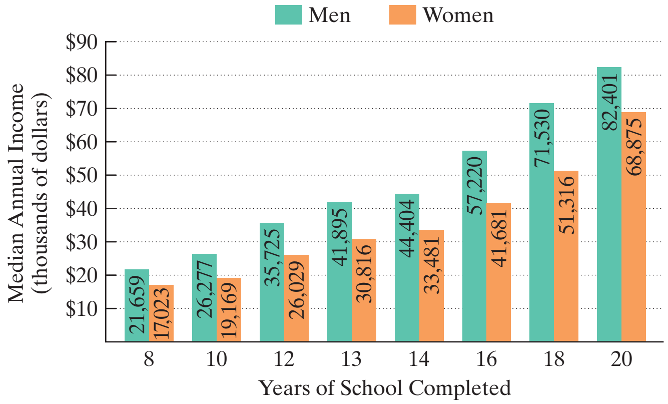
89. $\dfrac{(2x - 7)^5}{(2x - 7)^3}$

90. $\dfrac{(5x - 3)^6}{(5x - 3)^4}$

Application Exercises

As you complete more years of education, you can count on a greater income. The bar graph shows the median, or middlemost, annual income for Americans, by level of education, in 2004.

Median Annual Income, by Level of Education, 2004



Source: Bureau of the Census

Here are polynomial models that describe the median annual income for men, M, and for women, W, who have completed x years of education:

$$M = 177x^2 + 288x + 7075$$
$$W = 255x^2 - 2956x + 24{,}336$$

$$M = -18x^3 + 923x^2 - 9603x + 48{,}446$$
$$W = 17x^3 - 450x^2 + 6392x - 14{,}764.$$

Exercises 91–92 are based on these models and the data displayed by the graph.

91. a. Use the equation defined by a polynomial of degree 2 to find the median annual income for a man with 16 years of education. Does this underestimate or overestimate the median income shown by the bar graph? By how much?

b. Use the equations defined by polynomials of degree 3 to find a mathematical model for $M - W$.

c. According to the model in part (b), what is the difference in the median annual income between men and women with 14 years of education?

d. According to the data displayed by the graph, what is the actual difference in the median annual income between men and women with 14 years of education? Did the result of part (c) underestimate or overestimate this difference? By how much?

92. a. Use the equation defined by a polynomial of degree 2 to find the median annual income for a woman with 18 years of education. Does this underestimate or overestimate the median income shown by the bar graph? By how much?

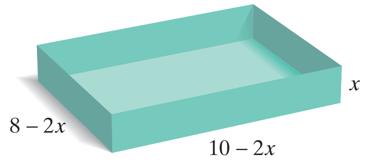
b. Use the equations defined by polynomials of degree 3 to find a mathematical model for $M - W$.

c. According to the model in part (b), what is the difference in the median annual income between men and women with 16 years of education?

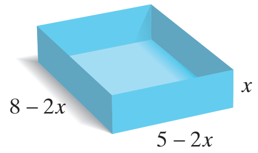
d. According to the data displayed by the graph, what is the actual difference in the median annual income between men and women with 16 years of education? Did the result of part (c) underestimate or overestimate this difference? By how much?

The volume, V, of a rectangular solid with length l, width w, and height h is given by the formula $V = lwh$. In Exercises 93–94, use this formula to write a polynomial in standard form that models, or represents, the volume of the open box.

93.

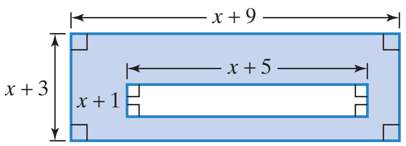


94.

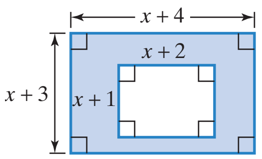


In Exercises 95–96, write a polynomial in standard form that models, or represents, the area of the shaded region.

95.



96.



Writing in Mathematics

97. What is a polynomial in x?

98. Explain how to subtract polynomials.

99. Explain how to multiply two binomials using the FOIL method. Give an example with your explanation.

100. Explain how to find the product of the sum and difference of two terms. Give an example with your explanation.

101. Explain how to square a binomial difference. Give an example with your explanation.

102. Explain how to find the degree of a polynomial in two variables.

Critical Thinking Exercises

Make Sense? *In Exercises 103–106, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

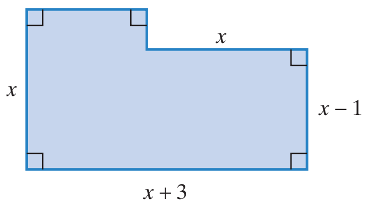
103. Knowing the difference between factors and terms is important: In $(3x^2y)^2$, I can distribute the exponent 2 on each factor, but in $(3x^2 + y)^2$, I cannot do the same thing on each term.

104. I used the FOIL method to find the product of $x + 5$ and $x^2 + 2x + 1$.

105. Many English words have prefixes with meanings similar to those used to describe polynomials, such as *monologue*, *binocular*, and *tricuspid*.

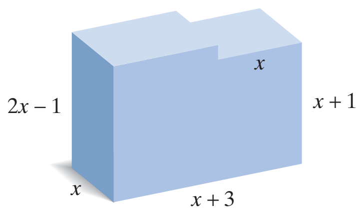
106. Special-product formulas have patterns that make their multiplications quicker than using the FOIL method.

107. Express the area of the plane figure shown as a polynomial in standard form.

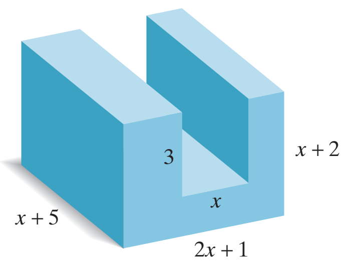


In Exercises 108–109, represent the volume of each figure as a polynomial in standard form.

108.



109.



110. Simplify: $(y^n + 2)(y^n - 2) - (y^n - 3)^2$.

Preview Exercises

Exercises 111–113 will help you prepare for the material covered in the next section. In each exercise, replace the boxed question mark with an integer that results in the given product. Some trial and error may be necessary.

111. $(x + 3)(x + \boxed{?}) = x^2 + 7x + 12$

112. $(x - \boxed{?})(x - 12) = x^2 - 14x + 24$

113. $(4x + 1)(2x - \boxed{?}) = 8x^2 - 10x - 3$