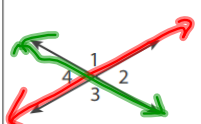


Lines & Angles

Vertical Angles

If two angles are vertical angles, then the angles are congruent.



$\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

$\angle 1 + \angle 4 = 180^\circ$
 $\angle 1 + \angle 2 = 180^\circ$

Linear Pair

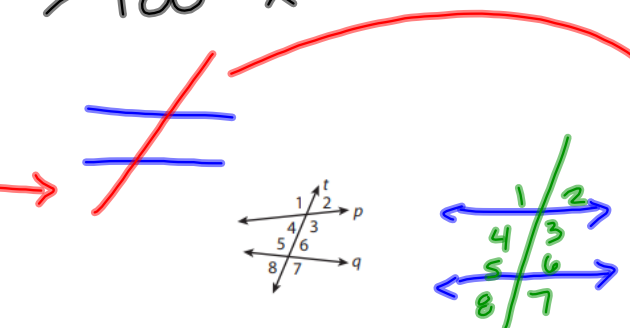


$\angle 1 + \angle 2 = 180$

Complementary angles
Supplementary angles

$\rightarrow 90^\circ - x$
 $\rightarrow 180^\circ - x$

Transversal



"If parallel lines are cut by a transversal...?"

Parallel

Angle Pair	Example
Corresponding angles lie on the same side of the transversal and on the same sides of the intersected lines.	$\angle 1$ and $\angle 5$
Same-side interior angles lie on the same side of the transversal and between the intersected lines.	$\angle 3$ and $\angle 6$
Alternate interior angles are nonadjacent angles that lie on opposite sides of the transversal between the intersected lines.	$\angle 3$ and $\angle 5$
Alternate exterior angles lie on opposite sides of the transversal and outside the intersected lines.	$\angle 1$ and $\angle 7$

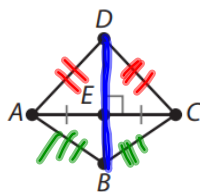
$\angle 1 \cong \angle 5$
 $\angle 3 + \angle 6 = 180$
 $\angle 3 \cong \angle 5$
 $\angle 1 \cong \angle 7$

Perpendicular

Perpendicular Bisector Theorem
 If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

Your Turn

Use the diagram shown. \overline{BD} is the perpendicular bisector of \overline{AC} .



- Suppose $ED = 16$ cm and $DA = 20$ cm. Find DC . =20
- Suppose $EC = 15$ cm and $BA = 25$ cm. Find BC . =25

Equations of Parallel & Perpendicular Lines

Parallel Lines

Have the SAME (equal) slopes

Example 1 Write the equation of each line in slope-intercept form.

- (A) The line parallel to $y = 5x + 1$ that passes through $(-1, 2)$

Parallel lines have equal slopes. So the slope of the required line is 5.

Use point-slope form.

Substitute for m, x_1, y_1 .

Simplify.

Solve for y .

The equation of the line is $y = 5x + 7$.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 5(x - (-1)) \Rightarrow y - 2 = 5(x + 1)$$

$$y - 2 = 5x + 5$$

$$y = 5x + 7$$

Perpendicular Lines

Slope is OPPOSITE RECIPROCAL

**Flip & Switch

- (B) The line perpendicular to $y = \frac{2}{5}x + 12$ that passes through $(-6, -8)$

The product of the slopes of perpendicular lines is -1 . So the slope of the required line is $\frac{5}{2}$

$y - y_1 = m(x - x_1)$ Use point-slope form.

$y - -8 = \frac{5}{2}(x - -6)$ Substitute for m, x_1, y_1 .

$y + 8 = \frac{5}{2}x + 15$ Simplify.

$y = \frac{5}{2}x + 7$ Solve for y .

The equation of the line is $y = \frac{5}{2}x + 7$

$$\frac{5}{2}(x + 6)$$

$$\frac{5}{2} \cdot \frac{3}{1}$$

4.1

Your Turn

- The measures of two vertical angles are 58° and $(3x + 4)^\circ$. Find the value of x .
- The measures of two vertical angles are given by the expressions $(x + 3)^\circ$ and $(2x - 7)^\circ$. Find the value of x . What is the measure of each angle?

Your Turn

You can represent the measures of an angle and its complement as x° and $(90 - x)^\circ$. Similarly, you can represent the measures of an angle and its supplement as x° and $(180 - x)^\circ$. Use these expressions to find the measures of the angles described.

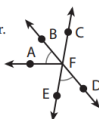
- The measure of an angle is equal to the measure of its complement.
- The measure of an angle is twice the measure of its supplement.

evaluate

Use this diagram and information for Exercises 1–4.

Given: $m\angle AFB = m\angle EFD = 50^\circ$

Points B, F, D and points E, F, C are collinear.



- Online Homework
- Hints and Help
- Extra Practice

- Determine whether each pair of angles is a pair of vertical angles, a linear pair of angles, or neither. Select the correct answer for each lettered part.

A. $\angle BFC$ and $\angle DFE$	<input type="radio"/> Vertical	<input type="radio"/> Linear Pair	<input type="radio"/> Neither
B. $\angle BFA$ and $\angle DFE$	<input type="radio"/> Vertical	<input type="radio"/> Linear Pair	<input type="radio"/> Neither
C. $\angle BFC$ and $\angle CFD$	<input type="radio"/> Vertical	<input type="radio"/> Linear Pair	<input type="radio"/> Neither
D. $\angle AFE$ and $\angle AFC$	<input type="radio"/> Vertical	<input type="radio"/> Linear Pair	<input type="radio"/> Neither
E. $\angle BFE$ and $\angle CFD$	<input type="radio"/> Vertical	<input type="radio"/> Linear Pair	<input type="radio"/> Neither
F. $\angle AFE$ and $\angle BFC$	<input type="radio"/> Vertical	<input type="radio"/> Linear Pair	<input type="radio"/> Neither
- Find $m\angle AFE$.
- Find $m\angle DFC$.

- The measure of an angle increased by 20° is equal to the measure of its complement.

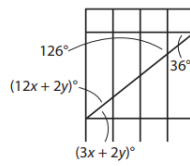
- Justify Reasoning** Complete the two-column proof for the theorem "If two angles are congruent, then their supplements are congruent."

Statements	Reasons
1. $\angle ABC \cong \angle DEF$	1. Given
2. The measure of the supplement of $\angle ABC = 180^\circ - m\angle ABC$.	2. Definition of the _____ of an angle
3. The measure of the supplement of $\angle DEF = 180^\circ - m\angle DEF$.	3. _____
4. _____	4. If two angles are congruent, their measures are equal.
5. The measure of the supplement of $\angle DEF = 180^\circ - m\angle ABC$.	5. Substitution Property of _____
6. The measure of the supplement of $\angle ABC =$ the measure of the supplement of $\angle DEF$.	6. _____
7. The supplement of $\angle ABC \cong$ the supplement of _____.	7. If the measures of the supplements of two angles are equal, then supplements of the angles are congruent.

4.2

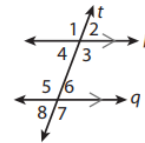
Your Turn

7. In the diagram of a gate, the horizontal bars are parallel and the vertical bars are parallel. Find x and y . Name the postulates and/or theorems that you used to find the values.



evaluate

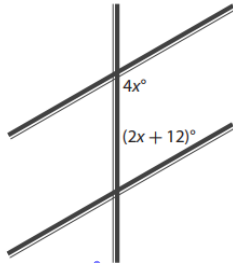
Use the figure to find angle measures. In the figure, $p \parallel q$.



- | | |
|---|---|
| 3. Suppose $m\angle 4 = 82^\circ$. Find $m\angle 5$. | 4. Suppose $m\angle 3 = 105^\circ$. Find $m\angle 6$. |
| 5. Suppose $m\angle 3 = 122^\circ$. Find $m\angle 5$. | 6. Suppose $m\angle 4 = 76^\circ$. Find $m\angle 6$. |
| 7. Suppose $m\angle 5 = 109^\circ$. Find $m\angle 1$. | 8. Suppose $m\angle 6 = 74^\circ$. Find $m\angle 2$. |

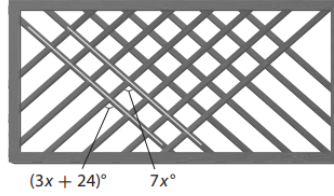
4.3

11. **Engineering** An overpass intersects two lanes of a highway. What must the value of x be to ensure the two lanes are parallel?



$$4x + 2x + 12 = 180$$

12. A trellis consists of overlapping wooden slats. What must the value of x be in order for the two slats to be parallel?

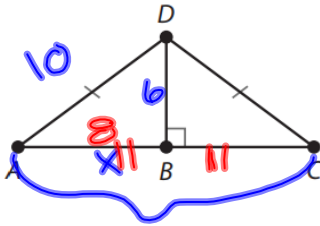


$$7x = 3x + 24$$

4.4

Your Turn

7. \overline{AD} is 10 inches long. \overline{BD} is 6 inches long. Find the length of \overline{AC} .

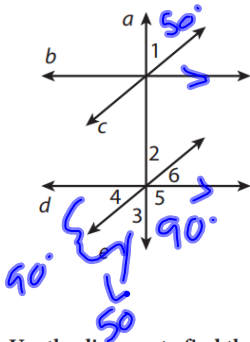


$$x^2 + 6^2 = 10^2$$

$$x = 8$$

Your Turn

9. Given: $b \parallel d$, $c \parallel e$, $m\angle 1 = 50^\circ$, and $m\angle 5 = 90^\circ$. Use the diagram to find $m\angle 4$.



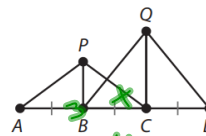
$$\angle 1 \cong \angle 3 \text{ Alt Ex}$$

$$= 40^\circ$$

$$\# 5 - 8$$

$$14 - 16$$

Use the diagram to find the lengths. \overline{BP} is the perpendicular bisector of \overline{AC} . \overline{CQ} is the perpendicular bisector of \overline{BD} . $AB = BC = CD$.



$$3^2 + 4^2 =$$

7. Suppose $AC = 12$ cm and $QD = 10$ cm. What is the length of \overline{QC} ? 8. Suppose $PB = 3$ cm and $AD = 12$ cm. What is the length of \overline{PC} ?

4.5

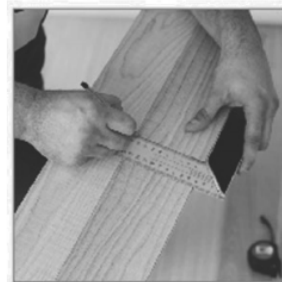
Your Turn

Write the equation of each line in slope-intercept form.

- 4. The line parallel to $y = -x$ that passes through $(5, 2.5)$
- 5. The line parallel to $y = \frac{3}{2}x + 4$ that passes through $(-4, 0)$

Reflect

- 6. A carpenter's square forms a right angle. A carpenter places the square so that one side is parallel to an edge of a board, and then draws a line along the other side of the square. Then he slides the square to the right and draws a second line. Why must the two lines be parallel?



Your Turn

Write the equation of each line in slope-intercept form.

- 7. The line perpendicular to $y = \frac{3}{2}x + 2$ that passes through $(3, -1)$
- 8. The line perpendicular to $y = -4x$ that passes through $(0, 0)$

Angles & Sides of a Triangle

Interior Angle

inside angle

SUM THEOREMS

The Triangle Sum Theorem

The sum of the angle measures of a triangle is 180° .

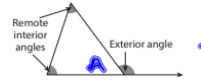
Polygon Angle Sum Theorem

The sum of the measures of the interior angles of a convex polygon with n sides is $(n - 2)180^\circ$.

Exterior Angle

An **exterior angle** is an angle formed by one side of a polygon and the extension of an adjacent side. Exterior angles form linear pairs with the interior angles.

A **remote interior angle** is an interior angle that is not adjacent to the exterior angle.



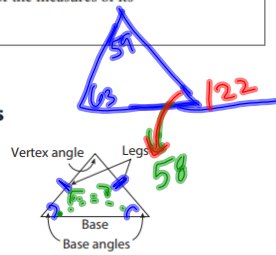
Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.

Isosceles Triangles

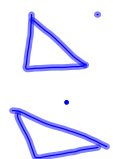
Explore Investigating Isosceles Triangles

- An **isosceles triangle** is a triangle with at least two congruent sides.
- The congruent sides are called the **legs** of the triangle.
- The angle formed by the legs is the **vertex angle**.
- The side opposite the vertex angle is the **base**.
- The angles that have the base as a side are the **base angles**.



Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the two angles opposite the sides are congruent.



Equilateral Triangles

- An **equilateral triangle** is a triangle with three congruent sides.
- An **equiangular triangle** is a triangle with three congruent angles.

Equilateral Triangle Theorem

If a triangle is equilateral, then it is equiangular.

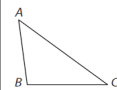
Converse of the Equilateral Triangle Theorem

If a triangle is equiangular, then it is equilateral.

Triangle Inequalities

Triangle Inequality Theorem

The sum of any two side lengths of a triangle is greater than the third side length.



$$AB + BC > AC$$

$$BC + AC > AB$$

$$AC + AB > BC$$

$\rightarrow AC > AB - BC$

$\rightarrow AC > BC - AB$

EX:

Given a triangle with sides 7, 12, and x , find the range of values for x .

According to the Triangle Inequality Theorem, the sum of any two side lengths of a triangle is greater than the third side length

$$7 + 12 > x \quad 7 + x > 12 \quad x + 12 > 7 \quad \text{Apply the Triangle Inequality Theorem.}$$

$$19 > x \quad x > 5 \quad x > -5 \quad \text{Simplify.}$$

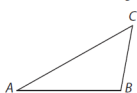
$$5 < x < 19 \quad \text{Combine the inequalities together.}$$

Side-Angle Relationships

****Angles "belong" to the side facing/opposite it**

Side-Angle Relationships in Triangles

If two sides of a triangle are not congruent, then the larger angle is opposite the longer side.



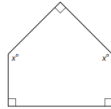
$$AC > BC$$

$$m\angle B > m\angle A$$

7.1

Your Turn

6. Determine the unknown angle measures in this pentagon $\rightarrow n=5$



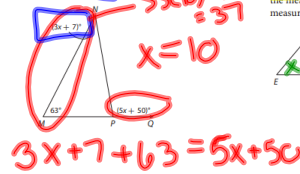
$180(n-2)$
 $3(90) + 2x = 540$

$n=4$
 $+\quad +\quad +\quad +\quad +\quad X = 360$

Your Turn

Determine the measure of the specified angle.

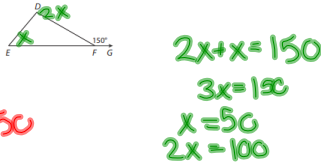
11. Determine $m\angle N$ in $\triangle MNP$.



8. Determine the unknown angle measures in a hexagon whose six angles measure $60^\circ, 108^\circ, 135^\circ, 210^\circ, 12^\circ$ and 216° .

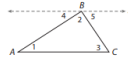
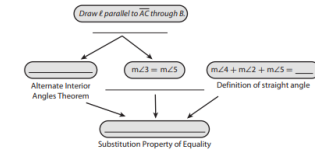
$n=6$
 $6-2 = 4 \cdot 180$
 $3b = 204$
 $b = 68$
 $2b = 136$

12. If the exterior angle drawn measures 150° , and the measure of $\angle D$ is twice that of $\angle E$, find the measure of the two remote interior angles.



2. Complete a flow proof for the Triangle Sum Theorem.

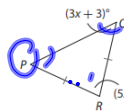
Given $\triangle ABC$
 Prove $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$



7.2

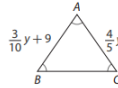
Your Turn

5. Find $m\angle P$.



$3(16) + 3 =$
 $\angle P = 3x+3$
 $2(3x+3) + 5x-2 = 180$
 $6x+6+5x-2 = 180$
 $11x+4 = 180$
 $11x = 176$
 $x = 16$

6. Katie's tutorial service is going so well that she is having shirts made with the equilateral triangle emblem. She has given the t-shirt company these dimensions. What is the length of each side of the triangle in centimeters?



$\frac{3}{10}y + 9 = \frac{4}{5}y - 1$
 $10(\frac{3}{10}y + 9) = 10(\frac{4}{5}y - 1)$
 $3y + 90 = 8y - 10$
 $-3y + 100 = -2y - 10$
 $100 = 5y$
 $y = 20$
 $\frac{4}{5}(20) - 1 = 16 - 1 = 15$

7.3

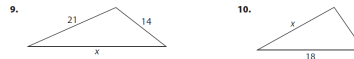
Your Turn

Determine if a triangle can be formed with the given side lengths. Explain your reasoning.

6. 12 units, 4 units, 17 units 7. 24 cm, 8 cm, 30 cm

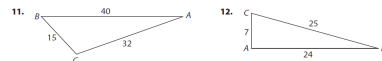
Your Turn

Find the range of values for x using the Triangle Inequality Theorem.



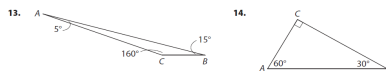
Your Turn

For each triangle, order its angle measures from least to greatest.



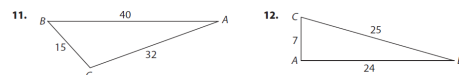
Your Turn

For each triangle, order the side lengths from least to greatest.



Your Turn

For each triangle, order its angle measures from least to greatest.



Special Segments in Triangles

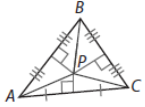
Point of Concurrency

Lines are concurrent if they intersect at the same point, called the point of concurrency (intersection)

Circumcenter

Circumcenter Theorem

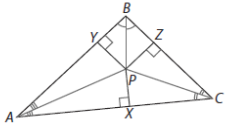
The perpendicular bisectors of the sides of a triangle intersect at a point that is equidistant from the vertices of the triangle.

$$PA = PB = PC$$


Incenter

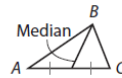
Incenter Theorem

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

$$PX = PY = PZ$$


Median

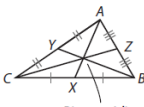
A segment whose endpoints are on 1) a vertex and 2) the midpoint of the opposite side



Centroid

Centroid Theorem

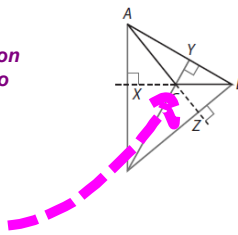
The centroid theorem states that the **centroid** of a triangle is located $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.



$$AP = \frac{2}{3} AX \quad BP = \frac{2}{3} BY \quad CP = \frac{2}{3} CZ$$

Altitude

A segment whose endpoints are on 1) a vertex and 2) perpendicular to the opposite side



Orthocenter

the point of intersection/concurrency of all three altitudes

Midsegment

A segment connected the midpoints of two sides of a triangle

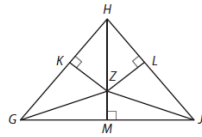
Triangle Midsegment Theorem

The segment joining the midpoints of two sides of a triangle is parallel to the third side, and its length is half the length of that side.

8.1

Your Turn

\overline{KZ} , \overline{LZ} , and \overline{MZ} are the perpendicular bisectors of $\triangle GHJ$. Copy the sketch and label the given information. Use that information to find the length of each segment. Note that the figure is not drawn to scale.



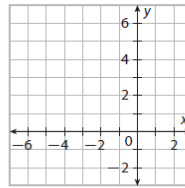
5. Given: $ZG = 65$, $HL = 63$, $ZL = 16$
Find: HJ and ZJ

6. Given: $ZM = 25$, $ZH = 65$, $GJ = 120$
Find: GM and ZG

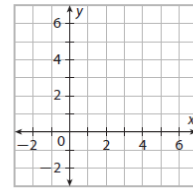
Your Turn

Graph the triangle with the given vertices and find the circumcenter of the triangle.

8. $Q(-4, 0)$, $R(0, 0)$, $S(0, 6)$



9. $K(1, 1)$, $L(1, 7)$, $M(6, 1)$

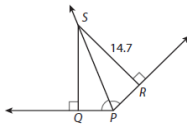


8.2

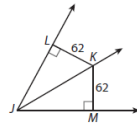
Your Turn

Find each measure.

4. $\angle SQP$



5. $m\angle LJM$, given that $m\angle KJM = 29^\circ$

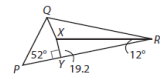


Your Turn

\overline{QX} and \overline{RX} are angle bisectors of $\triangle PQR$. Find each measure.

8. the distance from X to \overline{PQ}

9. $m\angle PQX$



8.3

Your Turn

6. Vertex L is 8 units from the centroid of $\triangle LMN$. Find the length of the median that has one endpoint at L .

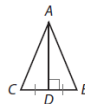
7. Let P be the centroid of $\triangle STU$, and let \overline{SW} be a median of $\triangle STU$. If $SW = 18$, find SP and PW .

Your Turn

Find the centroid of the triangles with the given vertices. Show your work and check your answer.

9. $P(-1, 7)$, $Q(9, 5)$, $R(4, 3)$

8. In $\triangle ABC$, the median \overline{AD} is perpendicular to \overline{BC} . If $AD = 21$ feet, describe the position of the centroid of the triangle.



10. $A(-6, 0)$, $B(0, 12)$, $C(6, 0)$

Your Turn

Find the orthocenter for the triangles described by each set of vertices.

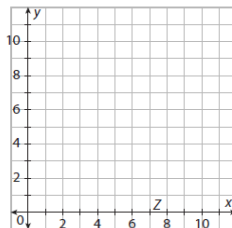
13. $Q(4, -3)$, $R(8, 5)$, $S(8, -8)$

14. $K(2, -2)$, $L(4, 6)$, $M(8, -2)$

8.4

Your Turn

4. The vertices of $\triangle XYZ$ are $X(3, 7)$, $Y(9, 11)$, and $Z(7, 1)$. U is the midpoint of \overline{XY} , and W is the midpoint of \overline{XZ} . Show that $\overline{UW} \parallel \overline{YZ}$ and $UW = \frac{1}{2}YZ$. Sketch $\triangle XYZ$ and \overline{UW} .



Your Turn

6. Find JL , PM , and $m\angle MLK$.

