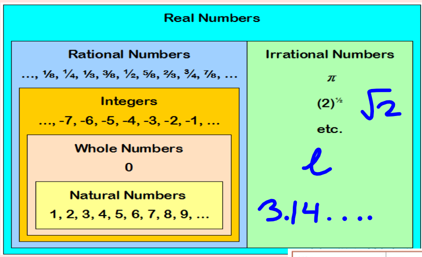
How are radicals & rational exponents related?

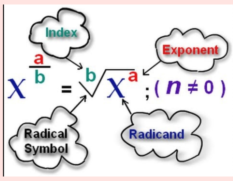
Real Number System



Irrational
a number that can't be a fraction; decimal that is non-terminating & non-repeating

Rational
a number that CAN be written as fraction
Rational usually refers TO fractions

Rational Exponents



$$x^{\frac{a}{b}} = \sqrt[b]{x^a};\ (n \neq 0)$$

(1) $\sqrt{x^3} = x^{\frac{3}{2}}$ (2) $\sqrt[3]{a^9} = a^{\frac{9}{3}} = a^3$ (3) $\sqrt[5]{2^{10}} = 2^{\frac{10}{5}} = 2^2 = 4$

$$\sqrt[\text{root}]{x^{\text{power}}} = x^{\frac{\text{power}}{\text{root}}}$$

TOP IN, BOTTOM OUT

Prime Factorization

Rewriting numbers as product of powers
Using Factor Trees!

EX: $32 = 2^5$

2, 16
2, 8
2, 4
2, 2

You Try! 243=

$36 = 2^2 \cdot 3^2$

2, 18
2, 9
3, 3

You Try! 200=

Your Turn

3. $8^{\frac{1}{3}}$
4. $16^{\frac{1}{2}} + 27^{\frac{1}{3}}$
5. $32^{\frac{3}{5}}$
6. $4^{\frac{5}{2}} - 4^{\frac{3}{2}}$

Exponent laws

The basic 3:

Law	Example
$x^0 = 1$	$3^0 = 1$
$x^1 = x$	$9^1 = 9$
$x^{-1} = 1/x$	$3^{-1} = 1/3$
$x^{-a} = 1/x^a$	$9^{-2} = 1/81$

The rest:

$x^a x^b = x^{a+b}$	$x^3 x^5 = x^8$
$x^a/x^b = x^{a-b}$	$x^{11}/x^4 = x^7$
$(x^a)^b = x^{ab}$	$(x^5)^3 = x^{15}$
$(xy)^a = x^a y^a$	$(xy)^4 = x^4 y^4$
$(x/y)^a = x^a/y^a$	$(x/y)^6 = x^6/y^6$

Your Turn

Simplify each expression. Assume all variables are positive.

5. $(x^2y)^2\sqrt[4]{y^4}$
6. $\dfrac{\sqrt[4]{x^8}}{\sqrt[4]{x^6}}$

8. $\left(\frac{1}{4x^4} \cdot x^{12}\right)^{-\frac{1}{2}}$
9. $\left(\frac{1}{9x^{12}} \cdot x^4\right)^{\frac{1}{2}}$

Answer:

How can you determine the result of operations on rational and irrational numbers?

Closure property

A set is closed under an operation if the result is always in that set

ex: are whole numbers closed under addition? ✓

are whole numbers closed under subtraction? ✗ $4-7=-3$

Sum

Rational & Rational= always rational

Irrational & Irrational= sometimes rational → additive inverses

Rational & Irrational= always irrational

$\sqrt{2}+\sqrt{2}$ = $\sqrt{2}-\sqrt{2}$ = . $\sqrt{2}+2$ = $2+2$ =

Product

Rational & Rational= always rational

Irrational & Irrational= sometimes rational

Rational & Irrational= always irrational → EXCEPT 0!

$\sqrt{2}\cdot\sqrt{2}$ = $\sqrt{2}\cdot\sqrt{3}$ = $2\cdot\sqrt{2}$ = $\sqrt{2}\cdot 0$ =

Which of the following expressions will have a rational product?

a. $\sqrt{4}\cdot\sqrt{9}$

b. $2\pi\cdot\frac{3}{4}$

c. $\frac{2}{3}\cdot(-4.5)$

d. $\frac{2}{5}\cdot\sqrt{5}$

e. $\sqrt{8}\cdot\sqrt{2}$

27. Determine whether each of the following is rational or irrational. Select the correct answer for each lettered part.

a. The product of $\sqrt{2}$ and $\sqrt{50}$ ◯ Rational ◯ Irrational

b. The product of $\sqrt{2}$ and $\sqrt{25}$ ◯ Rational ◯ Irrational

c. $C = 2\pi r$ evaluated for $r = \pi^{-1}$ ◯ Rational ◯ Irrational

d. $C = 2\pi r$ evaluated for $r = 1$ ◯ Rational ◯ Irrational

e. $A = 2\pi r^2$ evaluated for $r = \pi^{\frac{1}{2}}$ ◯ Rational ◯ Irrational

f. The product of $\sqrt{\frac{2}{\pi}}$ and $\sqrt{50\pi}$ ◯ Rational ◯ Irrational

g. The product of $\sqrt{2}$ and $\sqrt{\frac{9}{2}}$ ◯ Rational ◯ Irrational

Answer:

How do you write a geometric sequence?

Geometric Sequence	a pattern in which the terms are all multiplied by a constant....called the .....
Common Ratio	the constant ratio FOUND BY doing $\frac{\text{next term}}{\text{previous term}}$ 1, -1/3, 1/9, -1/27,... $r = -\frac{1}{3}$
Recursive Rule	2 things needed!! given f(1) good to find NEXT term, only if you know previous! $f(n) = f(n-1) \cdot r$ ex: $f(1)=1, \; f(n)=f(n-1)\left(-\frac{1}{3}\right)$
Explicit Rule	good to find any term $f(n) = f(1) \cdot r^{n-1}$ ex: $f(n) = 1\left(\frac{1}{3}\right)^{n-1}$

Your Turn

Write a recursive rule and an explicit rule for each geometric sequence.

5.

n	1	2	3	4	5
$f(n)$	7	14	28	56	112

6. Write a recursive rule and an explicit rule for the geometric sequence 128, 32, 8, 2, 0.5, ...

How to find the common ratio if given 2 terms

f(3)= 6 f(5)= 24 5-3 = 2.... soo

1) find common ratio *remember, if you went from f(1) to f(3), you're actually dividing to find r²*

$6 \cdot r^2 = \frac{24}{6} = 4$

$r = 2$

2) now plug in one of the points to find f(1)

$f(n) = f(1) \cdot \underline{2}^{\,n-1}$

$f(3) = f(1) \cdot 2^{3-1}$

$6 = f(1) \cdot \underline{2}^{\,3-1}$

$\frac{6}{4} = \frac{f(1) \cdot \cancel{2^2}}{\cancel{4}}$

$\frac{3}{2} = f(1)$

$f(n) = \frac{3}{2}(2)^{n-1}$

Write an explicit rule for the sequence using subscript notation.

11. The third term of a geometric sequence is $\frac{1}{27}$ and the fifth term is $\frac{1}{243}$. All the terms of the sequence are positive.

Answer:

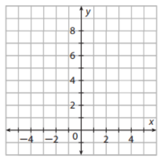
What is an exponential function and how do you write it?

Exponential Function

$y = ab^x$ $a \neq 0$ $b > 0$

Make a table for the function using the given domain. Then graph the function using the ordered pairs from the table.

4. $f(x) = 4\left(\frac{3}{2}\right)^x$; domain $= \{-3, -2, -1, 0, 1, 2\}$



a = y intercept

b= ratio

Domain & Range

x is always All Real Numbers

y is limited.... first by the y intercept

Writing Exponential Functions

find the y intercept & ratio!

from scenario

6. A piece of paper that is 0.2 millimeters thick is folded. Write an equation for the thickness t of the paper in millimeters as a function of the number n of folds.

a=

b=

from 2 points (or graph)

Write an equation for the function that includes the points.

7. $\left(-2, \frac{2}{5}\right)$ and $(-1, 2)$

1) Identify 2 points

2) Find the ratio

*3) plug in **b** and 1 of the points and solve for **a**!*

4) rewrite

CHECK IN!

22. Multipart Classification Determine whether each of the functions is exponential or not.

a. $f(x) = x^2$ ○ Exponential ○ Not exponential

b. $f(x) = 3 \cdot 2^x$ ○ Exponential ○ Not exponential

c. $f(x) = 3 \cdot \frac{1}{2}x$ ○ Exponential ○ Not exponential

d. $f(x) = 1.001^x$ ○ Exponential ○ Not exponential

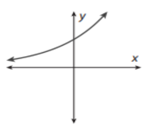
e. $f(x) = 2 \cdot x^3$ ○ Exponential ○ Not exponential

f. $f(x) = \frac{1}{10} \cdot 5^x$ ○ Exponential ○ Not exponential

End Behavior

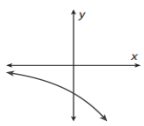
as $x \to \infty, y \to \infty$
as $x \to -\infty, y \to 0$

$a > 0$
$b > 1$



a. growth

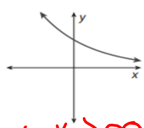
as $x \to \infty, y \to -\infty$.
as $x \to -\infty, y \to 0$



b.

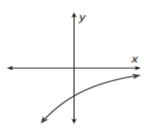
$a < 0$
$b > 1$

$a > 0$
$0 < b < 1$



c. decay

as $x \to \infty, y \to 0$
as $x \to -\infty, y \to \infty$



d.

$a < 0$
$0 < b < 1$

as $x \to \infty, y \to 0$
as $x \to -\infty, y \to -\infty$

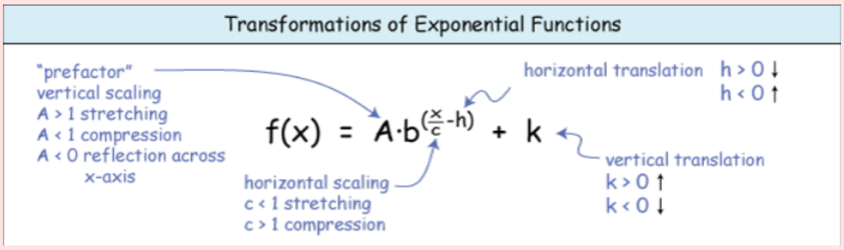
Answer:

How do you describe transformations of exponential functions?

Parent function

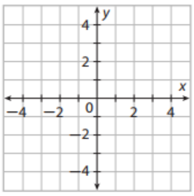
$$y = a(b)^x$$

Transformations:

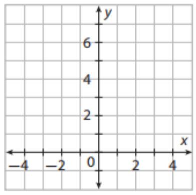


Graph each function, and describe the end behavior and find the *y*-intercept of each graph.

3. $f(x) = -0.5(1.5)^x$

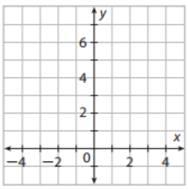


4. $f(x) = 4(1.5)^x$

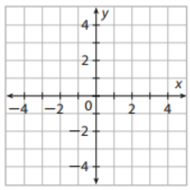


Graph each function, and describe its end behavior and *y*-intercept.

6. $f(x) = 2(0.6)^x$

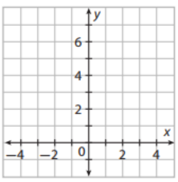


7. $f(x) = -0.25(0.6)^x$

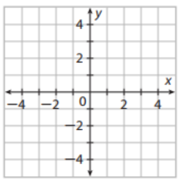


Graph the functions together on the same coordinate plane. Find the *y*-intercepts, and explain how they relate to the translation of the graph.

9. $f(x) = 0.4^x$ and $g(x) = 0.4^x + 4$



10. $f(x) = 2(1.5)^x$ and $g(x) = 2(1.5)^x - 3$



Answer:

How can you solve equations involving rational exponents?

Steps to solving:

1) Get b^x alone (use inverse operations)

2) Rewrite both sides so they have the same base (using powers)

3) If bases are the same, exponents are the same-- write equation using just exponents and solve

$$3(16^{2x}) = 12$$

$$16^{2x} = 4$$

$(4^2)^{2x} = 4$

$2(2x) = 1$

$4x = 1$

$x = 1/4$

$(2^4)^{2x} = 2^2$

$4(2x) = 2$

$8x = 2$

$x = 1/4$

Your Turn

Solve by equating exponents and using the Equality of Bases Property.

6. $\frac{2}{3}(3)^x = 18$

7. $\frac{3}{2}\left(\frac{4}{3}\right)^x = \frac{8}{3}$

U. $9^{3x+3} = 243^{x-2}$

G. $121^{3x+1} = 1331^x$

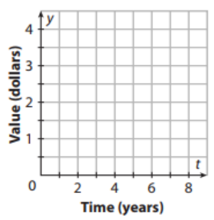
Answer:

How can you use an exponential function to model growth or decay?

Function	$y = a(b)^x$
Growth	$y = a(1+r)^t$
	*Remember to turn percentages into decimals!!
Decay	$y = a(1-r)^t$

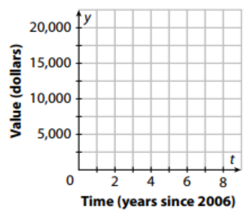
Your Turn

10. Write and graph an exponential growth function, and state the domain and range. Tell what the y-intercept represents. Sara sold a coin for \$3, and its value increases by 2% each year after it is sold. Find the value of the coin in 8 years.

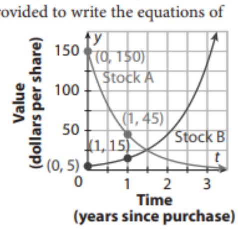


Your Turn

13. The value of a boat is depreciating at a rate of 9% per year. In 2006, the boat was worth \$17,800. Find the worth of the boat in 2013. Write an exponential decay function for this situation. Graph the function and state its domain and range. What does the *y*-intercept represent in the context of the problem?



15. The graph shows the value of two different shares of stocks over the period of 4 years since they were purchased. The values have been changing exponentially. Use the graphs provided to write the equations of the functions. Then describe and compare the behaviors of both functions.



Answer:

How can you rewrite exponential equations to show different features?

Manipulating an existing expression

Getting the variable alone & changing ratio

The population of a colony of bacteria doubles every 8 hours. If the number of bacteria starts at 80, the population P after t hours is given by $P(t) = 80 \bullet 2^{\frac{t}{8}}$. What is an equivalent form of $P(t)$ that shows the approximate hourly growth factor for the bacteria population?

a. $P(t) = 80 \bullet 1.091^t$
b. $P(t) = 80 \bullet 1.125^t$
c. $P(t) = 80 \bullet 1.25^t$
d. $P(t) = 80 \bullet 1.414^t$

"Changing" an existing expression

Multiplying by a number (and it's inverse) to change the ratio

On September 8, 2009, the price of gold reached $1000 per ounce. Over the next year, the price of an ounce of gold can be modeled by $y = 1000(1.255)^t$ where t is years after September 8, 2009. Find the approximate monthly percent increase in the price of gold.
Round your answer to the nearest hundredth of a percent.

a. 3.83%
b. 1.91%
c. 2.13%
d. 8.33%

A certain radioactive element has a half-life of 35 days. If you had 100 grams of this element, the mass m of the element after t 35-day intervals is represented by $m = 100(0.5)^t$. Find the approximate daily decay rate of this element.

a. 98.0%
b. 17.5%
c. 2.0%
d. 1.4%

For the period 1880–1970, an exponential regression model that gives the population P of Dallas, Texas, t decades after 1880 is $P(t) = 18,800(1.58)^t$. Transform the model to find the annual growth rate. Explain your reasoning, and show your work. By approximately what percent was the population of Dallas growing every 10 years? Every year?

Answer: