What makes 2 figures similar?

Dilation

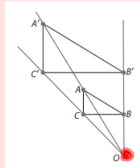
Properties of:

Properties of Dilations

- Dilations preserve angle measure.
- Dilations preserve betweenness.
- Dilations preserve collinearity.
- Dilations preserve orientation.
- Dilations map a line segment (the pre-image) to another line segment whose length is the product of the scale factor and the length of the pre-image.
- Dilations map a line not passing through the center of dilation to a parallel line and leave a line passing through the center unchanged.

Center of Dilation

The **center of dilation** is the fixed point about which all other points are transformed by a dilation. The ratio of the lengths of corresponding sides in the image and the preimage is called the **scale factor**.



Scale Factor

$\frac{\text{new}}{\text{old}}$

Similar

A **similarity transformation** is a transformation in which an image has the same shape as its pre-image. Similarity transformations include reflections, translations, rotations, and dilations. Two plane figures are **similar** if and only if one figure can be mapped to the other through one or more similarity transformations.

Circle Similarity Theorem

All circles are similar!

Properties of Similar Figures

Properties of Similar Figures

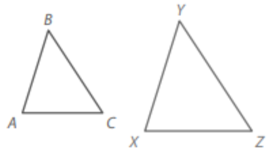
Corresponding angles of similar figures are congruent.

Corresponding sides of similar figures are proportional.

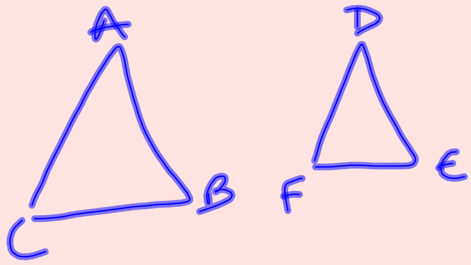
If $\triangle ABC \sim \triangle XYZ$, then

$\angle A \cong \angle X \quad \angle B \cong \angle Y \quad \angle C \cong \angle Z$

$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$



AA Triangle Similarity Theorem



$\angle C \cong \angle F$, $\angle A \cong \angle D$, ABC similar DEF

SSS Triangle Similarity

if 3 sides of 1 Δ are proportional to 3 sides of Δ, the Δ are similar

SAS Triangle Similarity

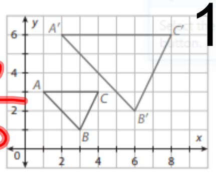
→ 2 sides & included angle

11.1 p. 582

4. Is the scale factor of the dilation of $\triangle ABC$ equal to $\frac{1}{2}$? Explain.

· distance
· (points)

$\frac{A'C'}{AC}$ $\frac{6}{3} = 2$



B'

5. Square A is a dilation of square B.
What is the scale factor?

a. $\frac{1}{7}$

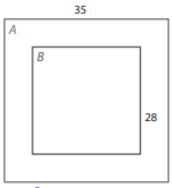
b. $\frac{4}{5}$

→ **c.** $\frac{5}{4}$

d. 7

e. $\frac{25}{16}$

$\frac{35}{28}$



Determine if the transformation of figure A to figure B on the coordinate plane is a dilation. Verify ratios of corresponding side lengths for a dilation.

→ A'

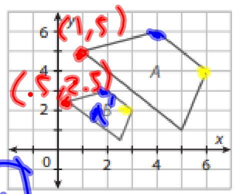
→ is center of dilation (0,0)?

12. 2

(10,6) (5,3) (4,1) (8,2)

$\frac{(3,8)}{(1.5,4)}$

13.

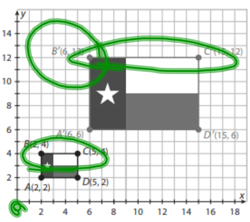


$\frac{(2,3)}{(4,6)}$ $\frac{(3,2)}{(6,4)}$

$\frac{1}{2}$

16. You work at a photography store. A customer has a picture that is 4.5 inches tall. The customer wants a reduced copy of the picture to fit a space of 1.8 inches tall on a postcard. What scale factor should you use to reduce the picture to the correct size?

17. Computer Graphics An artist uses a computer program to enlarge a design, as shown. What is the scale factor of the dilation?



$\frac{(6,12)}{(2,4)} = 3$

$\frac{9}{3} = 3$

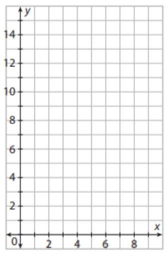
H.O.T. Focus on Higher Order Thinking

19. Draw $\triangle DEF$ with vertices D (3, 1) E (3, 5) F (0, 5).

a. Determine the perimeter and the area of $\triangle DEF$.

b. Draw an image of $\triangle DEF$ after a dilation having a scale factor of 3, with the center of dilation at the origin (0, 0). Determine the perimeter and area of the image.

c. How is the scale factor related to the ratios $\frac{\text{perimeter } \triangle D'E'F'}{\text{perimeter } \triangle DEF}$ and $\frac{\text{area } \triangle D'E'F'}{\text{area } \triangle DEF}$?

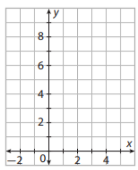


20. Draw $\triangle WXY$ with vertices (4, 0), (4, 8), and (−2, 8).

a. Dilate $\triangle WXY$ using a factor of $\frac{1}{4}$ and the origin as the center. Then dilate its image using a scale factor of 2 and the origin as the center. Draw the final image.

b. Use the scale factors given in part (a) to determine the scale factor you could use to dilate $\triangle WXY$ with the origin as the center to the final image in one step.

c. Do you get the same final image if you switch the order of the dilations in part (a)? Explain your reasoning.

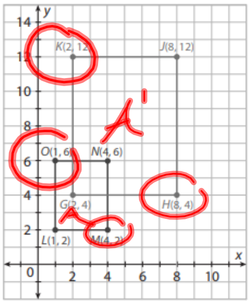


11.2

Your Turn

Determine whether the two figures are similar using similarity transformations. Explain.

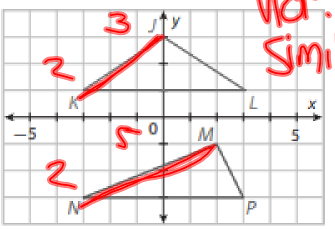
3. *LMNO* and *GHJK*



$\frac{(2,12)}{(1,6)}$ $(x,y)\rightarrow(2x,2y)$

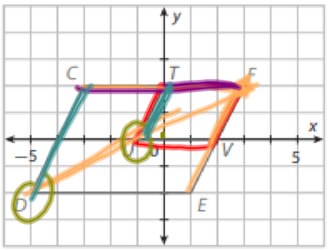
$\frac{(8,4)}{(4,2)}$

4. $\triangle JKL$ and $\triangle MNP$



not similar

5. *CDEF* and *TUVF* → dilated w/ a center of F



~~$\frac{U}{D} = \frac{(-1,0)}{(-5,-2)}$~~ doesn't work

look @ distance

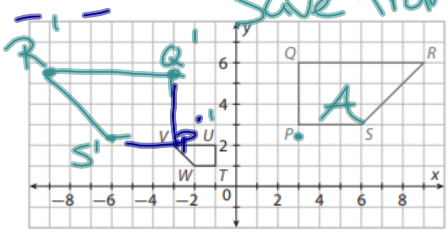
$\frac{TF}{CF} = \frac{3}{6} = \frac{1}{2}$ $\frac{TU}{CD} = \frac{\sqrt{1^2+2^2}}{\sqrt{2^2+4^2}}$ $\frac{\sqrt{5}}{\sqrt{20}} = \frac{1}{2}$

Your Turn

For each pair of similar figures, find a sequence of similarity transformations that maps one figure to the other. Use coordinate notation to describe the transformations.

8. *PQRS* to *TUVW*

Save translations for end



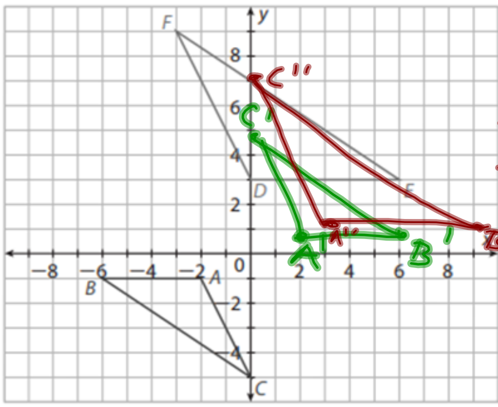
→ reflection across y → $(x,y)\rightarrow(-x,y)$

→ dilation → $(x,y)\rightarrow(\frac{1}{3}x,\frac{1}{3}y)$ centered @ (0,0)

$\frac{new}{old}$ → $\frac{T}{P'}$ $\frac{(-1,1)}{(-3,3)} = \frac{1}{3}$

$\frac{TU}{PQ} = \frac{1}{3}$ $\frac{UV}{QR} = \frac{2}{6} = \frac{1}{3}$

9. $\triangle ABC$ to $\triangle DEF$



$\frac{DE}{AB} = \frac{3}{2}$

• rotation 180° → $(x,y)\rightarrow(-x,-y)$

• dilation → $(x,y)\rightarrow(\frac{3}{2}x,\frac{3}{2}y)$ (1.5x,

• translation? → $(x,y)\rightarrow(x-3,y+1.5)$

Reflect

11.3

4. If you know two figures are similar, what angle or side measurements must you know to find the dilation used in the transformations mapping one figure to another?

Your Turn

5. Triangles $\triangle PQR$ and $\triangle LMN$ are similar. If $QR = 6$ and $MN = 9$, what similarity transformation (in coordinate notation) maps $\triangle PQR$ to $\triangle LMN$?

$\frac{9}{6} = \frac{3}{2} = 1.5$

$(x, y) \to (1.5x, 1.5y)$

6. Error Analysis Triangles $\triangle DEF$ and $\triangle UVW$ are similar. $\frac{DE}{UV} = \frac{VW}{EF}$ Is the statement true?

$\frac{\text{new}}{\text{old}}$ $\frac{UV}{DE} = \frac{VW}{EF}$

Explain 2 Applying Properties of Similar Figures

The properties of similar figures can be used to find the measures of corresponding parts.

Example 2 **Given that the figures are similar, find the values of x and y.**

Ⓐ

Find the value of x.

$\angle C \cong \angle R$, so $m\angle C = m\angle R$

$4x + 27 = 95$

$4x = 68$

$x = 17$

Find the value of y.

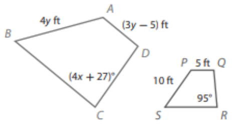
$\frac{AB}{PS} = \frac{AD}{PQ}$

$\frac{4y}{10} = \frac{3y - 5}{5}$

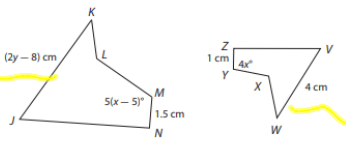
$\frac{4y}{10} \cdot 10 = \frac{3y - 5}{5} \cdot 10$

$4y = 6y - 10$

$y = 5$



Ⓑ



Find the value of x.

$m\angle LMN = m\angle XYZ$

$5(x-5) = 4x$

Find the value of y.

$\frac{JK}{VW} = \frac{MN}{ZY}$

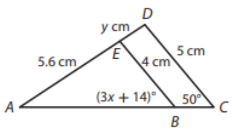
$\frac{2y-8}{4} = \frac{1.5}{1}$

$1(2y-8) = 4(1.5)$

$2y - 8 = 6$

Your Turn

Use the diagram, in which $\triangle ABE \sim \triangle ACD$.



$AD = AE + ED$

$5.6 + y$

8. Find the value of x.

$\angle B \cong \angle C$

$3x + 14 = 50$

9. Find the value of y.

$\frac{AD}{AE} = \frac{DC}{EB}$

$\frac{5.6 + y}{5.6} = \frac{5}{4}$

$4(5.6 + y) = 5(5.6)$

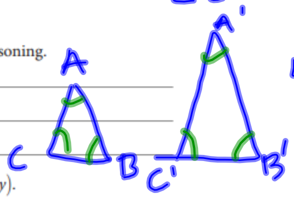
$22.4 + 4y = 28$

$4y = 5.6$

$y = 1.4$

Elaborate

10. Consider two similar triangles $\triangle ABC$ and $\triangle A'B'C'$. If both $m\angle A' = m\angle C$ and $m\angle B' = m\angle A$, what can you conclude about triangle $\triangle ABC$? Explain your reasoning.



11. Rectangle $JKLM$ maps to rectangle $RSTU$ by the transformation $(x, y) \to (4x, 4y)$. If the perimeter of $RSTU$ is x, what is the perimeter of $JKLM$ in terms of x?

$\frac{1}{4}x$

12. Essential Question Check-In If two figures are similar, what can we conclude about their corresponding parts?

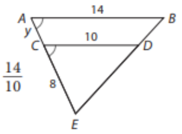
11.4

Reflect

4. In Example 2A, is there another way you can set up the proportion to solve for *BE*?

5. **Discussion** When asked to solve for *y*, a student sets up the proportion as shown. Explain why the proportion is wrong. How should you adjust the proportion so that it will give the correct result?

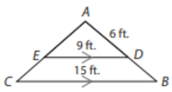
$\frac{y}{8} = \frac{14}{10}$



$$\frac{AB}{CD} = \frac{AE}{CE} \rightarrow y+8$$

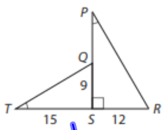
Your Turn

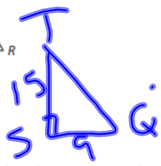
6. A builder was given a design plan for a triangular roof as shown. Explain how he knows that $\triangle AED \sim \triangle ACB$. Then find *AB*.



7. Find *PQ*, if possible.

$$\frac{SR}{SQ} = \frac{PS}{TS} \rightarrow PS = PQ + 9$$



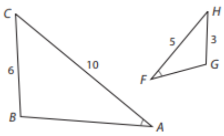


$$\frac{12}{9} = \frac{x}{15}$$

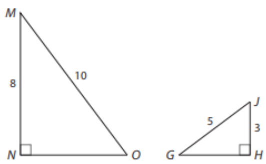
Your Turn

If possible, determine whether the given triangles are similar. Justify your answer.

10.



11.



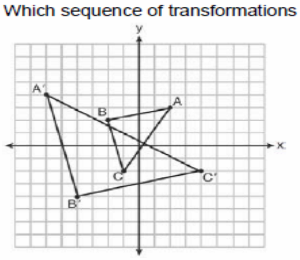
A triangle is dilated by a scale factor of 3 with the center of dilation at the origin. Which statement is true?

A. The area of the image is nine times the area of the original triangle.
B. The perimeter of the image is nine times the perimeter of the original triangle.
C. The slope of any side of the image is three times the slope of the corresponding side of the original triangle.
D. The measure of each angle in the image is three times the measure of the corresponding angle of the original triangle.

The point $(3, -2)$ is rotated 90° about the origin and then dilated by a scale factor of 4. What are the coordinates of the resulting image?

A. $(-12, 8)$
B. $(12, -8)$
C. $(8, 12)$
D. $(-8, -12)$

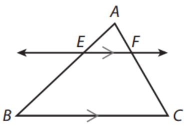
Which sequence of transformations will map $\triangle ABC$ onto $\triangle A'B'C'$?



A. reflection and translation
B. rotation and reflection
C. translation and dilation
D. dilation and rotation

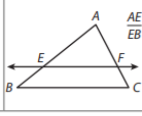
How can you find the point in a direct line segment that partitions the given segment ina given ratio?

Triangle Proportionality Theorem

Theorem	Hypothesis	Conclusion
If a line parallel to a side of a triangle intersects the other two sides, then it divides those sides proportionally.	 $\overleftrightarrow{EF} \parallel \overline{BC}$	$\frac{AE}{EB} = \frac{AF}{FC}$

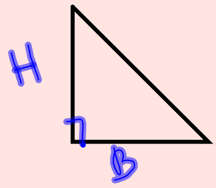
Converse

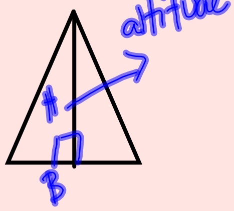
Converse of the Triangle Proportionality Theorem

Theorem	Hypothesis	Conclusion
If a line divides two sides of a triangle proportionally, then it is parallel to the third side.	 $\frac{AE}{EB} = \frac{AF}{FC}$	$\overleftrightarrow{EF} \parallel \overline{BC}$

Area of Triangle

1/2 BH





Directed Line Segment

segment between 2 points with specific direction

Partitions

To divide

To partition a directed line segment:

1) Write the ratio of point A to point B that expresses the distance of point along the segment

--write as PART to WHOLE

$a \text{ to } b \rightarrow \frac{a}{a+b}$ $\left\{ 5 \text{ to } 3 \rightarrow \frac{5}{5+3} = \frac{5}{8} \right.$

2) Calculate the Rise & Run from point A to point B

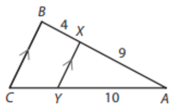
3) Find the value of the Rise and Run using the ratio that **partitions** the segment!

4) Locate the point P that corresponds to the Rise and Run

Explain 2 Applying the Triangle Proportionality Theorem

Example 2 **Find the length of each segment.**

Ⓐ $\overline{CY}$



It is given that $\overline{XY} \parallel \overline{BC}$ so $\frac{AX}{XB} = \frac{AY}{YC}$ by the Triangle Proportionality Theorem.

Substitute 9 for AX, 4 for XB, and 10 for AY.

Then solve for CY.

$\frac{9}{4} = \frac{10}{CY}$

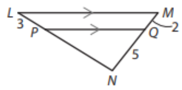
Take the reciprocal of both sides.

$\frac{4}{9} = \frac{CY}{10}$

Next, multiply both sides by 10.

$10\left(\frac{4}{9}\right) = \left(\frac{CY}{10}\right)10 \quad \rightarrow \quad \frac{40}{9} = CY, \text{ or } 4\frac{4}{9} = CY$

Ⓑ Find PN.



It is given that $\overline{PQ} \parallel \overline{LM}$, so $\frac{NQ}{QM} =$ ______ by the Triangle Proportionality Theorem.

Substitute ____ for NQ, ____ for QM, and 3 for ____.

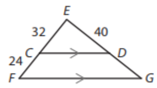
$\frac{5}{2} = \frac{NP}{3}$

Multiply both sides by ____: $\square\left(\frac{5}{2}\right) = \square\left(\frac{NP}{3}\right) \rightarrow$ ____________ $= NP$

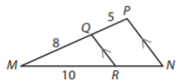
Your Turn

Find the length of each segment.

5. $\overline{DG}$

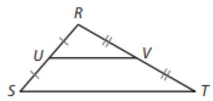


6. $\overline{RN}$



Reflect

7. **Critique Reasoning** A student states that $\overline{UV}$ must be parallel to $\overline{ST}$. Do you agree? Why or why not?

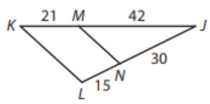


Explain 4 Applying the Converse of the Triangle Proportionality Theorem

You can use the Converse of the Triangle Proportionality Theorem to verify that a line is parallel to a side of a triangle.

Example 4 **Verify that the line segments are parallel.**

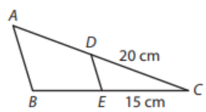
Ⓐ $\overline{MN}$ and $\overline{KL}$



$\frac{JM}{MK} = \frac{42}{21} = 2 \qquad \frac{JN}{NL} = \frac{30}{15} = 2$

Since $\frac{JM}{MK} = \frac{JN}{NL}$, $\overline{MN} \parallel \overline{KL}$ by the Converse of the Triangle Proportionality Theorem.

Ⓑ $\overline{DE}$ and $\overline{AB}$ (Given that $AC = 36$ cm, and $BC = 27$ cm)



$AD = AC - DC = 36 - 20 = 16$

$BE = BC - \square = \square - \square = \square$

$\frac{CD}{DA} = \frac{\square}{\square} = \frac{\square}{\square} \qquad \frac{CE}{EB} = \frac{\square}{\square} = \square$

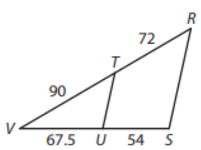
Since $\frac{CD}{DA} =$ ——, $\overline{DE} \parallel \overline{AB}$ by the ________________________ Theorem.

Reflect

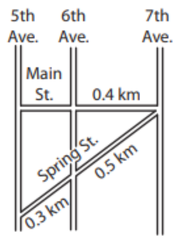
8. **Communicate Mathematical Ideas** In $\triangle ABC$, in the example, what is the value of $\frac{AB}{DE}$? Explain how you know.

Your Turn

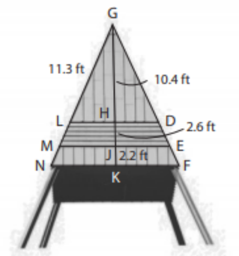
9. Verify that $\overline{TU}$ and $\overline{RS}$ are parallel.



11. On the map, 5th Avenue, 6th Avenue, and 7th Avenue are parallel. What is the length of Main Street between 5th Avenue and 6th Avenue?



12. **Multi-Step** The storage unit has horizontal siding that is parallel to the base.



a. Find LM.

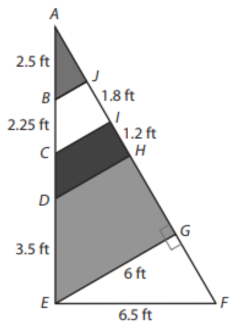
b. Find GM.

c. Find MN to the nearest tenth of a foot.

d. **Make a Conjecture** Write the ratios $\frac{LM}{MN}$ and $\frac{HJ}{JK}$ as decimals to the nearest hundredth and compare them. Make a conjecture about the relationship between parallel lines $\overleftrightarrow{LD}$, $\overleftrightarrow{ME}$, and $\overleftrightarrow{NF}$ and transversals $\overleftrightarrow{GN}$ and $\overleftrightarrow{GK}$.

Lesson Performance Task

Shown here is a triangular striped sail, together with some of its dimensions. In the diagram, segments BJ, CI, and DH are all parallel to segment EG. Find each of the following:



1. AJ
2. CD
3. HG
4. GF
5. the perimeter of $\triangle AEF$
6. the area of $\triangle AEF$
7. the number of sails you could make for $10,000 if the sail material costs $30 per square yard