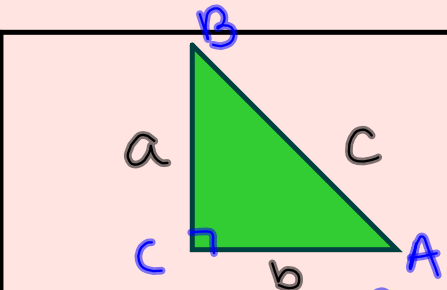
How can you use the trigonometric ratios in calculations involving right triangles?

To find the ratio of a trig function:



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Tangent

opposite side / adjacent side

$\tan A^\circ = \frac{a}{b}$

Sine

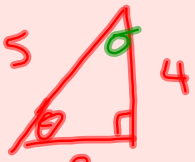
opposite / hypotenuse

$\sin A^\circ = \frac{a}{c}$

Cosine

adjacent / hypotenuse

$\cos A^\circ = \frac{b}{c}$



$\theta + \sigma = 90$

$\frac{4}{3}$ $\frac{3}{4}$

$\frac{4}{5}$ $\frac{3}{5}$

$\frac{3}{5}$ $\frac{4}{5}$

complementary angles

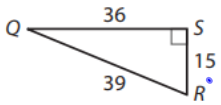
tangents → reciprocal

$\sin A = \cos B$

$\sin(x) = \cos(90 - x)$

Your Turn

Find the tangent of each specified angle. Write each ratio as a fraction and as a decimal rounded to the nearest hundredth.



5. $\angle Q$ **6.** $\angle R$

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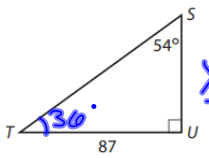
$\frac{15}{36} = \frac{5}{12} \approx 0.42$ $\frac{36}{15} = \frac{12}{5} \approx 2.2$

Ⓑ For right triangle $\triangle STU$, what is the length of the leg adjacent to $\angle S$?



$x \approx 63.2$

Step 1 Write a tangent ratio that involves the unknown length.

$$\tan S = \frac{\text{length of leg}}{\text{length of leg}} \quad \frac{\angle S}{\text{to } \angle S} = \frac{\square}{\square}$$

Step 2 Identify the given values and substitute into the tangent equation.

Given: $TU =$ ☐ and $m\angle S =$ ☐°

Substitute: tan ☐° $= \frac{\square}{SU}$

$\tan 54^\circ = \frac{87}{x}$

Step 3 Solve for the unknown leg length.

Multiply both sides by SU, then divide both sides by 54°. $SU = \frac{\square}{\square^\circ}$

Use a calculator to find 54° and substitute. $SU \approx \frac{\square}{\square}$

Divide. Round to the nearest tenth. $SU \approx$ ☐

$x \tan 54^\circ = 87$

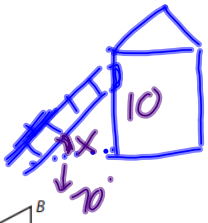
$x = \frac{87}{\tan 54^\circ}$

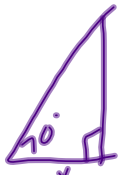
$x = 63.2$ $\frac{87}{1.376}$

Your Turn

7. A ladder needs to reach the second story window, which is 10 feet above the ground, and make an angle with the ground of 70°. How far out from the building does the base of the ladder need to be positioned?

3.6 ft





$3.6 = x$

$\tan 70^\circ = \frac{10}{x}$

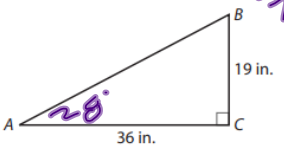
$x = \frac{10}{\tan 70^\circ}$

$x = 3.63$

Example 3 **Find the measure of the indicated angle. Round to the nearest degree.**

Ⓐ What is $m\angle A$?

$\tan A = \frac{19}{36}$



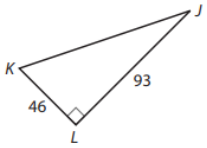
Step 1 Write the tangent ratio for $\angle A$ using the known values.	Step 2 Write the inverse tangent equation.	Step 3 Evaluate using a calculator and round as indicated.
$\tan A = \frac{19}{36}$	$\tan^{-1}\frac{19}{36} = m\angle A$	$m\angle A \approx 27.82409638 \approx 28^\circ$

Ⓑ What is $m\angle B$?

Step 1 Write the tangent ratio for $\angle B$ using the known values.	Step 2 Write the inverse tangent equation.	Step 3 Evaluate using a calculator and round as indicated.
$\tan B =$ ☐/☐	$\tan^{-1}\frac{36}{19} = m\angle B$	$m\angle B \approx$ ☐° $\approx 62^\circ$

Your Turn

8. Find $m\angle J$.



$\tan J = \frac{46}{93}$ $\tan^{-1}\left(\frac{46}{93}\right)$

$m\angle J = 26.3$

5. In a right triangle $\triangle PQR$ with $PR = 5$, $QR = 3$, and $m\angle Q = 90°$, what are the values of sin P and cos P?

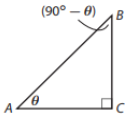
$\sin P =$ ☐

$\cos P =$ ☐

Trigonometric Ratios of Complementary Angles

If $\angle A$ and $\angle B$ are the acute angles in a right triangle, then $\sin A = \cos B$ and $\cos A = \sin B$.

Therefore, if θ ("theta") is the measure of an acute angle, then $\sin \theta = \cos(90° - \theta)$ and $\cos \theta = \sin(90° - \theta)$.



Your Turn

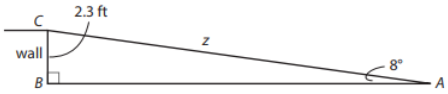
Write each trigonometric expression.

8. Given that $\cos 73° \approx 0.292$, write the sine of a complementary angle.

9. Given that $\sin 45° \approx 0.707$, write the cosine of a complementary angle.

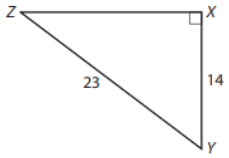
Your Turn

11. Suppose a new regulation states that the maximum angle of a ramp for wheelchairs is 8°. At least how long must the new ramp be? Round to the nearest tenth of a foot.



Your Turn

Find the acute angle measures in $\triangle XYZ$, to the nearest degree.



13. $m\angle Y$

14. $m\angle Z$

How can you find the missing angle of a right triangle?

To find the ratio of a trig function

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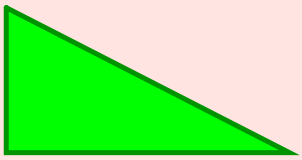
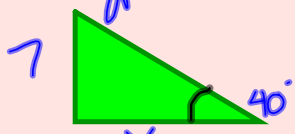
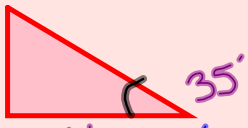


$\sin = \frac{opp}{hyp}$ $\cos = \frac{adj}{hyp}$ $\tan = \frac{opp}{adj}$

To find a missing side:

solve for x!





Examples:

$x(\tan 40^\circ) = \left(\frac{7}{x}\right)x$

$x\tan 40^\circ = 7$

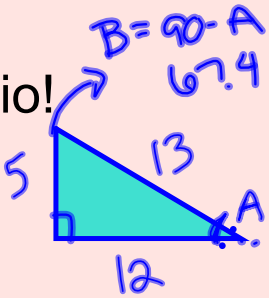
$x = \frac{7}{\tan 40^\circ} = 8.3$

$11(\tan 35^\circ) = \left(\frac{x}{11}\right)11$

$11\tan 35^\circ = x = 7.7$

To find a missing angle

**You do the inverse of the ratio!



$\tan^{-1}\frac{5}{12} \approx 22.6$

$\cos^{-1}\frac{12}{13} \approx 22.6$

$\sin^{-1}\frac{5}{13} \approx 22.6$

Relationship between ratios:

The ratios of complementary angles are RECIPROCALS (tangent)

Tan:

ie, $\tan A = \frac{1}{\tan B}$ $\sin 72 = x$. $\cos B = x$

Sin & Cos

$\sin A = \cos B$ $\sin A = \cos(90 - A)$

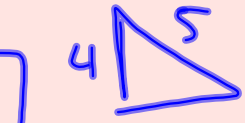
$\sin B = \cos A$ $\cos A = \sin(90 - A)$ "complementary"

Pythagorean Triple

$a^2 + b^2 = c^2$

positive integers make up all sides of right triangle

ex: 3, 4, 5	5,12,13
7, 24, 25	8,15,17

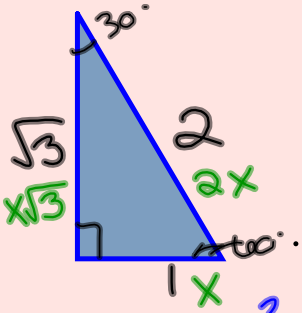


$3^2+4^2=5^2$

Special Right Triangles

30-60-90

$1-\sqrt{3}-2$



$\sin 30^\circ = \frac{1}{2}$ $\quad \sin 60^\circ = \frac{\sqrt{3}}{2}$

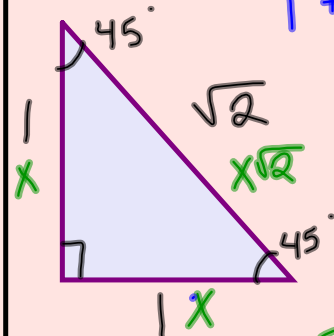
$\cos 30^\circ = \frac{\sqrt{3}}{2}$ $\quad \cos 60^\circ = \frac{1}{2}$

$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\quad \tan 60^\circ = \frac{\sqrt{3}}{1}$

$1^2+1^2=2$

$(\sqrt{2})^2$

45-45-90



$\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 45^\circ = 1$

$x^2+x^2=\sqrt{2x^2}=x\sqrt{2}$

x^2 $\quad x^1$

$\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=\frac{\sqrt{3}}{3}$

$\sqrt{3\cdot 3}$

$\sqrt{9}$