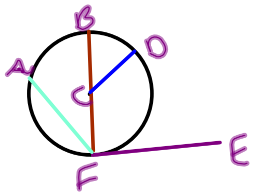
Angle and Segment Relationships in Circles

Vocab:



chord → $\overline{BF}$, $\overline{AF}$

radius → $\overline{CD}$

tangent → $\overleftrightarrow{EF}$

Central Angle

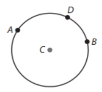
measure of a central angle is EQUAL to its intercepted arc

Arc

Arc Addition Postulate

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$m\widehat{ADB} = m\widehat{AD} + m\widehat{DB}$

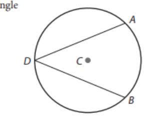


Inscribed Angle

Inscribed Angle Theorem

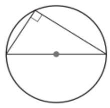
The measure of an inscribed angle is equal to half the measure of its intercepted arc.

$m\angle ADB = \frac{1}{2} m\widehat{AB}$



Inscribed Angle of a Diameter Theorem

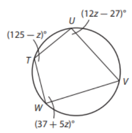
The endpoints of a diameter lie on an inscribed angle if and only if the inscribed angle is a right angle.



Inscribed Quadrilaterals

Inscribed Quadrilateral Theorem

If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.



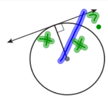
$\angle U + \angle W = 180$

$12z - 27 = 37 + 5z$

Tangent Relationships

Tangent-Radius Theorem

If a line is tangent to a circle, then it is perpendicular to a radius drawn to the point of tangency.

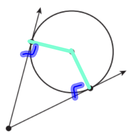


And the converse!

Circumscribed Angle

Circumscribed Angle Theorem

A circumscribed angle of a circle and its associated central angle are supplementary.

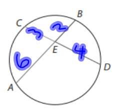


15.3

Chord-Chord Product

Chord-Chord Product Theorem

If two chords intersect inside a circle, then the products of the lengths of the segments of the chords are equal.



$AE \cdot EB = CE \cdot ED$

$3 \cdot 4 = 6 \cdot 2$

$12 = 12$

15.4

Intersecting Chords

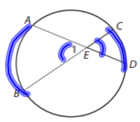
The Intersecting Chords Angle Measure Theorem

If two secants or chords intersect in the interior of a circle, then the measure of each angle formed is half the sum of the measures of its intercepted arcs.

Chords $\overline{AD}$ and $\overline{BC}$ intersect at E.

$m\angle 1 = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$

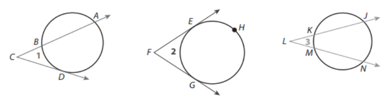
$\frac{1}{2}$ sum



Tangent-Secant

The Tangent-Secant Exterior Angle Measure Theorem

If a tangent and a secant, two tangents, or two secants intersect in the exterior of a circle, then the measure of the angle formed is half the difference of the measures of its intercepted arcs.



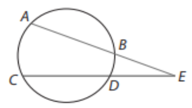
$m\angle 1 = \frac{1}{2}(m\widehat{AD} - m\widehat{BD})$ $m\angle 2 = \frac{1}{2}(m\widehat{EHG} - m\widehat{EG})$ $m\angle 3 = \frac{1}{2}(m\widehat{JN} - m\widehat{KM})$

$\frac{1}{2}$ difference

Additional Important Segment Relationships!

Secant-Secant Product Theorem

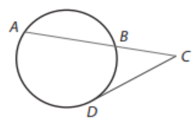
If two secants intersect in the exterior of a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.



$AE \cdot BE = CE \cdot DE$

Secant-Tangent Product Theorem

If a secant and a tangent intersect in the exterior of a circle, then the product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared.

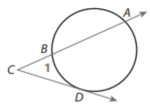


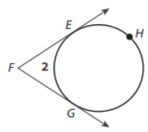
$AC \cdot BC = DC^2$

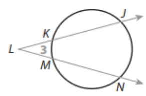
Tangent-Secant

The Tangent-Secant Exterior Angle Measure Theorem

If a tangent and a secant, two tangents, or two secants intersect in the exterior of a circle, then the measure of the angle formed is half the difference of the measures of its intercepted arcs.







$m\angle 1 = \frac{1}{2}(m\widehat{AD} - m\widehat{BD})$

$m\angle 2 = \frac{1}{2}(m\widehat{EHG} - m\widehat{EG})$

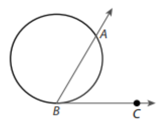
$m\angle 3 = \frac{1}{2}(m\widehat{JN} - m\widehat{KM})$

The Tangent-Secant Interior Angle Measure Theorem

If a tangent and a secant (or a chord) intersect on a circle at the point of tangency, then the measure of the angle formed is half the measure of its intercepted arc.

Tangent $\overrightarrow{BC}$ and secant $\overrightarrow{BA}$ intersect at B.

$m\angle ABC = \frac{1}{2} m\widehat{AB}$



How can you write the equation of a circle given the radius and center?

Radians --> degrees

$x \cdot \frac{180}{\pi}$

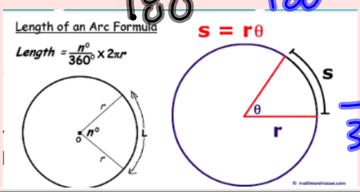
$1 \text{ radian (or 1 rad)} = \frac{180^\circ}{\pi}$

Degrees--> radians

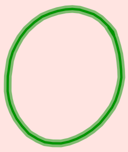
$\theta^\circ \cdot \frac{\pi}{180}$ $\quad \frac{\theta \pi}{180}$

$1 \text{ degree (or } 1^\circ) = \frac{\pi}{180^\circ}$

Arc Length



$\frac{m^\circ}{360} \cdot 2\pi r$ $\quad 360^\circ = 2\pi \text{ rad}$



Sector Area

Area of Sector

$A = \frac{\theta}{360^\circ}\pi r^2$ where θ is in degrees

$A = \frac{1}{2}r^2\theta$ where θ is in radians

$\frac{m^\circ}{360} \cdot \pi r^2$

Equation of a circle

$(x-h)^2 + (y-k)^2 = r^2$

r = radius (h,k)= center

$(x-2)^2 + (y-3)^2 = 9$
$\downarrow$
3^2

center $(2,3)$
$r=3$

$(x-1)^2 + (y+3)^2 = 25$ $\quad \sqrt{25}$

$c: (1,-3)$ $\quad r=5$

Arc Length & Sector Area

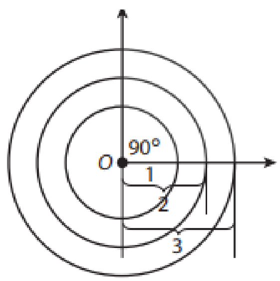
Area πr^2

Circumference $\pi d \overset{\text{or}}{=} 2\pi r$

Arc Length $\rightarrow \frac{m}{360} \cdot 2\pi r$

Sector Area $\longrightarrow \frac{m}{360} \cdot \pi r^2$

Concentric Circles

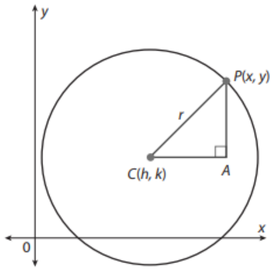


ratio of arc length = ratio of radii

Equation of a Circle

Equation of a Circle

The equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.



Example 1 **Write the equation of the circle with the given center and radius.**

(A) Center: $(-2, 5)$; radius: 3

$(x - h)^2 + (y - k)^2 = r^2$	Write the general equation of a circle.
$(x - (-2))^2 + (y - 5)^2 = 3^2$	Substitute -2 for h, 5 for k, and 3 for r.
$(x + 2)^2 + (y - 5)^2 = 9$	Simplify.

(B) Center: $(4, -1)$; radius: $\sqrt{5}$

$(x - h)^2 + (y - k)^2 = r^2$	Write the general equation of a circle.
$(x - (\square))^2 + (y - (\square))^2 = \square^2$	Substitute $\square$ for h, $\square$ for k, and $\square$ for r.
$(x \square)^2 + (y \square)^2 = \square$	Simplify.

Completing the Square

**Used when equation isn't given IN the circle format

(A) $x^2 - 4x + y^2 + 2y = 20$

Step 1 Complete the square twice to write the equation in the form $(x - h)^2 + (y - k)^2 = r^2$.

$x^2 - 4x + (\ )^2 + y^2 + 2y + (\ )^2 = 20 + (\ )^2$	Set up to complete the square.
$x^2 - 4x + \left(\frac{-4}{2}\right)^2 + y^2 + 2y + \left(\frac{2}{2}\right)^2 = 20 + \left(\frac{-4}{2}\right)^2 + \left(\frac{2}{2}\right)^2$	Add $\left(\frac{-4}{2}\right)^2$ and $\left(\frac{2}{2}\right)^2$ to both sides.
$x^2 - 4x + 4 + y^2 + 2y + 1 = 20 + 5$	Simplify.
$(x - 2)^2 + (y + 1)^2 = 25$	Factor.

Step 2 Identify h, k, and r to determine the center and radius.

$h = 2$ $\quad k = -1$ $\quad r = \sqrt{25} = 5$

So, the center is $(2, -1)$ and the radius is 5.

Step 3 Graph the circle.

- Locate the center of the circle.
- Place the point of your compass at the center
- Open the compass to the radius.
- Use the compass to draw the circle.